Analytical solutions for evaluating CPTu soundings in overconsolidated Hartford clay

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ABSTRACT: Two closed-form analytical solutions are applied to piezocone penetration test (CPTu) data in stiff overconsolidated clay from Hartford, Connecticut to evaluate total stress and effective stress parameters. These include: (a) limit plasticity solution for assessing the effective stress friction angle ($\phi'$); (b) hybrid cavity expansion-critical state model to interpret the operational rigidity index ($I_R = G/su$), undrained shear strength ($su$), and yield stress ratio ($YSR = \sigma_y'/\sigma_{vo'}$), where $G$ = shear modulus and $\sigma_y'$ = preconsolidation stress. Data from laboratory triaxial compression tests (drained and undrained) and oedometer tests from a nearby bridge are shown to support the CPTu interpretations.

Keywords: clays; cone penetration; friction angle; piezocone; yield stress

1. Introduction

During geotechnical site investigations, cone penetration tests (CPT), or more specifically, piezocone tests (CPTu) provide three continuous readings with depth: cone tip resistance ($q_t$), sleeve friction ($f_s$), and penetration porewater pressure ($u_2$). In addition to the direct readings, various net values can be utilized including: net cone resistance ($q_{net} = q_t - \sigma_{vo}$), excess porewater pressure ($\Delta u_2 = u_2 - u_0$), and effective cone resistance ($q_{e} = q_t - u_2$), where $\sigma_{vo}$ = total vertical overburden stress and $u_0$ = hydrostatic porewater pressure. Herein, a new parameter termed $\Delta u_0 = (u_2 - \sigma_{vo})$ is derived and utilized for piezocone interpretation.

Soil classification by CPTu is achieved indirectly, using either one or more approaches: (a) rules of thumb, (b) soil behavior type charts, or (c) probabilistic methods [1, 2].

When CPT soundings encounter clay soils, the practicing geotechnical engineer most always starts an evaluation centered on total stress parameters, i.e., the evaluation of the undrained shear strength ($s_u$). In the classic approach, this is sought using a bearing capacity expression:

$$s_u = \frac{q_{net}}{N_{kt}}$$

where $N_{kt}$ is a cone bearing factor [3, 4, 5]. The magnitude of $N_{kt}$ can be ascertained from theoretical, analytical, or numerical solutions, however, its value if more often assumed from experience, such as a mean value $N_{kt} = 12$ (range: $8.6 \leq N_{kt} \leq 17.3$) for soft clays tested in triaxial compression mode, while other means and ranges can be assigned for simple shear or vane shear.

It is well appreciated that soil behavior is controlled by effective stress states, not total stress. Therefore, a more appropriate evaluation of CPTu data in clays is via effective stress analyses. Two closed-form analytical solutions are available for this purpose: (a) an effective stress limit plasticity solution for $\phi'$ developed at NTH and (b) a hybrid cavity expansion-critical state model for YSR, $s_u$, and $I_R$. Brief reviews of these approaches are provided in the following sections.

1.1. NTH solution for obtaining $\phi'$

Over four decades ago, an effective stress limit plasticity solution was developed for the evaluation of effective friction angle of sands, silts, and clays from in-situ CPTu soundings at the Norwegian Institute of Technology (NTH), now the Norwegian University of Science & Technology (NTNU) in Trondheim [6, 7, 8, 9, 10]. For the case of non-cemented soils with an effective cohesion intercept $c' = 0$ and an angle of plastification $\beta = 0$, the solution is closed form and given by:

$$Q' = \frac{\tan^2(45^\circ + \phi'/2) \cdot \exp(\sigma \cdot \tan \phi') - 1}{1 + 6 \cdot \tan \phi'(1 + \tan \phi') \cdot \beta}$$

where $Q' = \text{normalized net cone tip resistance adjusted for stress history effects} = Q/\text{YSR}$, $\text{YSR} = \sigma_y'/\sigma_{vo'}$ = yield stress ratio (i.e., OCR = overconsolidation ratio), $Q = q_{net}/\sigma_{vo}$; $\Lambda = 1-C_v/C_s$ = plastic volumetric strain ratio, $C_v$ = swelling or recompression index, $C_s$ = virgin compression index, and $B_q = \Delta u_2/q_{net}$ = normalized porewater pressure parameter [11, 12].

Typical values of $\Lambda$ are around 0.7 for triaxial compression and 0.8 for direct simple shear for inorganic natural clays of low sensitivity, while for sensitive and quick clays, $\Lambda = 1$.

A graphical representation of Eq. (1) is presented in Figure 1 where $\phi'$ is shown in terms of $Q'$ and $B_q$. Approximations are given for $\phi'$ directly for two cases: (a) intact normally consolidated (NC) to overconsolidated (OC) clays where the following ranges apply: $0.05 \leq B_q \leq 1.0$ and $18^\circ \leq \phi' \leq 45^\circ$; and (b) fissured overconsolidated clays where $B_q = 0$.

For intact NC to OC clays:

$$\phi' \approx 29.5^\circ \cdot B_q^{0.121} \cdot [0.256 + 0.336 \cdot B_q + \log Q']$$

Figure 1
while for the fissured OC clays [12]:

$$\phi' = 8.18 \cdot \ln(2.13 \cdot Q')$$ (4)

The NTH solution has been calibrated by comparing the $$\phi'$$ obtained from triaxial compression tests with the value of $$\phi'$$ from piezocone data using several database studies, including: (a) chamber tests [13]; (b) centrifuge test series [14], and natural clay deposits [12, 15]. In the chamber tests and centrifuge series, artificial clays were prepared, often using either kaolin or a kaolin-sand or kaolinitic-silica slurry. Data from a total of some 145 clays have been collected to validate this approach.

### 1.3. Cone bearing factor

The SCE-CSSM solution is based on the classical assessment of undrained shear strength via Eq. (1) where $$N_{kt}$$ is the cone bearing value obtained by Vesič [21] and corresponds to the triaxial compression mode ($$s_{uk}$$):

$$N_{kt} = \frac{4}{3} \cdot [\ln(IR)+1]+\frac{\pi}{2}+1$$ (11)

### 1.4. Simplified expressions

For a first-order estimation of yield stress or preconsolidation stress, simplified expressions can be obtained by taking characteristic values of the geoparameters, including $$\phi' = 30^\circ$$ (or $$M_c = 1.2$$), $$I_k = 100$$, and $$\Lambda = 1$$. For "normal" clays that are uncemented, inorganic, and relatively insensitive [19, 20]:

$$\sigma_{pc} = 0.33 \cdot q_{net} = 0.54 \cdot U_2 = 0.60 \cdot q_E$$ (12)

These expressions are also useful in screening clays soils to identify organic soils, the following hierarchy is noted [22, 23]:

$$0.54 \cdot U_2 < 0.33 \cdot q_{net} < 0.60 \cdot q_E$$ (13)

They can also be utilized to identify when clays are highly sensitive or quick, given by the following order [24, 25, 26]:

$$0.60 \cdot q_E < 0.33 \cdot q_{net} < 0.54 \cdot U_2$$ (14)

If organic clays are found, a modified Eq.(12) for $$q_{net}$$ is used [22, 23], whereas if sensitive clays are identified,
then a modified SCE-CSSM set of expressions is available [26].

Figure 2. Site location in East Hartford, Connecticut

2 Case study from Hartford, Connecticut

The aforementioned analytical CPTu solutions for evaluating $\phi'$, $I_R$, YSR, and $s_u$ in clays are applied to piezocone test results in overconsolidated red-brown to gray varved clay in Hartford, Connecticut. The silty clay is a glacio-lacustrine deposit from the ancient Lake Hitchcock.

2.1 Site Conditions

The project involved the foundation design for a new 5-story hotel in downtown Hartford on the east side of the Connecticut River (see Figure 2). Prior geotechnical studies for review included soil borings and laboratory tests for interstate highway construction of I-84 and I-91 and the nearby Bissell Bridge, plus some additional information from the Charter Oaks and William H. Putnam Bridges [27]. General subsurface conditions at these sites include a sand layer over a varved clay underlain by glacial till.

2.2 Laboratory parameters

Results from laboratory one-dimensional consolidation and triaxial compression tests on the varved silty clay have been reported for the highway construction research projects. Natural water contents of the clay ranged between $49\% < w_n < 66\%$ with corresponding void ratios: $1.3 < e_0 < 1.9$, and unit weights: $15.5 < \gamma_t < 17.1$ kN/m$^3$. Figure 3 presents the profiles of effective yield stress ($\sigma_p'$) and consolidation indices ($C_c$, $C_s$, and $\lambda$) from the lab tests on the clay [27]. The consolidation tests showed an average value of yield stress difference (YSD = $\sigma_p' - \sigma_u$) of about YSD = 253 kPa in the clay, indicating a low to moderate degree of overconsolidation.

A series of undrained CIUC and drained CIDC triaxial tests were performed on the varved clay. A summary of results from the Bissett Bridge are presented in Figure 4, indicating: $c' = 0$ and $\phi' = 24^\circ$.

2.3 Soil borings

In a preliminary geotechnical exploration for the hotel, a series of 6 soil test borings were conducted, however no energy measurements were available for the standard penetration tests (SPT). Uncorrected N-values in the varved silty clay ranged from 2 to 5 blows/0.3 meters. Therefore, in the final geotechnical study, a series of three seismic piezocone tests were performed.

2.4 Piezocone soundings

A representative seismic piezocone test (SCPTu) at the site is presented in Figure 5. The results show the profiles of $q_t$, $f_s$, and $u_2$, plus the downhole mode of shear wave velocity ($V_s$) at the site. While the CPT readings are taken at approximate 25-mm intervals, the $V_s$ measurements are procured at 1-m intervals.
At this test location, the sounding encountered an upper layer of sand 11.5 m thick overlying a firm clay stratum that extends to about 26 m depth. The groundwater table lies 3.5 m below grade.

3 Piezocone interpretation of Hartford clay

Results of the SCPTu sounding will be evaluated using a first-order estimate and screening, then the applied application of the NTH and SCE-CSSM solutions.

3.1 Preliminary CPT evaluation

For a first-order evaluation of the soil conditions, a simplified approach to the CPT interpretations was attained via the assessment of yield stress from net cone resistance and soil behavior type [3, 20]. The general relationship is given by:

$$\sigma'_{p} = 0.33 \cdot (q_{net})^{m'} \cdot (\sigma_{am} / 100)^{1-m'}$$

(15)

where the exponent $m'$ varies from 1.00 in intact clays to 0.72 in clean quartz sands. In fact, for un cemented, inorganic clays of low-medium sensitivity, the exponent $m'$ generally tracks with the CPT material index, $I_c$:

$$I_c = 1 - \left( \frac{I_e}{2.65} \right)^{25}$$

(16)

where the CPT material index is given by [2, 3]:

$$I_e = \sqrt{(3.47 - \log Q_{am})^2 + (1.22 + \log F_{r})^2}$$

(17)

For the East Hartford site, Figure 6 shows the derived profiles of $I_c$ and first-order evaluation of $\sigma'_{p}$ with depth. The $I_c$ profile clearly shows the sand layer (zone 6) with mixed silty sandy soils (zone 5) to 8.5 m depths underlain by clay (zone 3) to around 26 m.

The profile of $\sigma'_{p}$ indicates a YSR $\approx 2$ for the varved clay, whereas slightly higher values of 2.5 $<\text{YSR} < 3$ are indicated by the consolidation tests. The difference can be easily explained by reference to equation (11) where a nominal value of $\phi' = 30^\circ$ has been used for the general case, whereas the Hartford varved silty clay has an actual characteristic value $\phi' = 24^\circ$.

3.2 Screening for clay soil type

Following the screening procedures given previously in Section 1.4, the application of Eq. (12) is presented in Figure 7 and indicates that the Hartford clay can be considered "normal", and not in the grouping of organic clays per Eq. (13), nor in the category of sensitive and structured clays given by Eq. (14).

3.3 Rigidity index of Hartford clay

The rigidity index of the clay can be found from any of the three expressions, Eq. (8), (9), or (10). Using Eq. (9), for example, a plot of $q_{net}$ versus $q_{E}$ provides the slope $a_y = 1.73$, as indicated by Figure 8. Using an effective stress friction angle $\phi' = 24^\circ$ ($M_c = 0.94$) gives an operational rigidity index $I_R = 132$.

Similarly, a plot of $\Delta u$ vs. $q_{net}$ gives a slope $a_x = 0.427$. This in turn with $M_c = 0.94$ and Eq. (8) provides $I_R = 143$. Likewise, a plot of $\Delta u$ vs. $q_{E}$ provides $a_y = 0.727$. From Eq. (10), the value of $I_R = 132$ is obtained. More or less, the three expressions all provide an operational value of $I_R$ in a narrow range, between 132 and 143. In general, the expressions given for slope parameters $a_x$ and $a_y$ tend to be more stable and reliable than that given by slope $a_z$.

An independent assessment of rigidity index is afforded through a derivation by Krage et al. [28]. Here the
results of $V_s$ are necessary since the small-strain shear modulus ($G_0 = G_{\text{max}}$) is required, where:

$$G_0 = G_{\text{max}} = \rho \cdot (V_s)^2$$  \hspace{1cm} (18)

where $\rho = \gamma / g_0$ = total mass density of the soil, $\gamma$ = total soil unit weight, and $g_0 = 9.8 \text{ m/s}^2$ = gravitational constant.

Soil unit weight (kN/m$^3$) can be evaluated from $V_s$ (m/s) and corresponding depth $z$ (in meters) using [1]:

$$\gamma = 8.32 \cdot \log(V_s) - 1.61 \cdot \log(z)$$  \hspace{1cm} (19)

A means of estimating $I_R$ in clays at 50% mobilized strength is given by [28]:

$$I_{R50} = \frac{G_0}{(q_{\text{net}})^{0.75} \left(\sigma_{\text{vo}}\right)^{0.25}}$$  \hspace{1cm} (20)

where $G_0$, $q_{\text{net}}$, and $\sigma_{\text{vo}}$ are given in the same consistent units. Figure 9 shows the comparison of the $I_R$ value from Eq. (20) in good agreement with that from the SCE-CSSM solution given by Eq. (9).

![Figure 9. Undrained rigidity index from SCPTu data in clay](image)

### 3.4 Friction angle of clay from CPTu

The CPTu can directly provide the effective friction angle of Hartford clay by plotting $q_{\text{net}}$ and $\Delta u_2$ versus the equivalent stress, $\sigma'_e = \sigma_{\text{vo}} \cdot \text{OCR}$, as seen in Figure 10 [12]. The corresponding normalized piezocone values obtained are: $Q' = 2.77$ and $U'_2 = 1.64$, giving $B'_q = 0.592$. Using either the rigorous Eq. (2) or approximate formula given by Eq. (3), or chart solution of Figure 1, a CPT-determined value of $\phi' = 24.8^\circ$ is obtained for this clay. This compares well with the CIUC and CIDC data in Figure 4.

Notably, results from lab tests on varved clay from nearby Amherst test site in Massachusetts also determined $\phi' = 25^\circ$ from CIUC test series and $\phi' = 24^\circ$ from consolidated drained direct shear box tests [29, 30].

For evaluating the value of $\phi'$ in clean to silty sands, the following equation applies [1, 3, 19]:

$$\phi' = 17.6^\circ + 11.0 \cdot \log(\varphi_m)$$  \hspace{1cm} (21)

![Figure 10. Post-processing of CPTu data in OC varved Hartford clay for evaluation of effective friction angle](image)

### 3.5 Yield stress profiles of Hartford clay

Three yield stress profiles can now be produced for the overconsolidated varved clay from the CPTu records, via Eqs. (5), (6), and (7) and input parameters: $M_c = 0.94$ ($\phi' = 24^\circ$), $I_R = 132$, and $\Lambda = 0.9$. Figure 12 presents these plots showing very good agreement with the yield stresses interpreted from the consolidation test results. The YSR profile decreases from around 3.5 at a depth of 12 m to around 2.5 at 25 m.

### 3.6 Profile of undrained shear strength

The assessment of the undrained shear strength is provided by the cone bearing factor $N_{kt}$ from Eq. (11) using $I_R = 132$ and Eq. (1) with $q_{\text{net}}$. The value of $N_{kt} = 10.4$ with this approach and applies to the triaxial...
compression mode. For benchmark values, either the SHANSEP approach or CSSM can be employed to determine $s_{uc}$ from the stress history data. For the SHANSEP approach [29]:

$$S_{nc} \sigma_{vo} = S \cdot OCR^n$$

(22)

where $S = (s_{uc}/\sigma_{vo})_{NC}$, $OCR = overconsolidation\ ratio$ (i.e., YSR), and $m = empirical\ exponent$. DeGroot and Lutenegger [30] report values of $S = 0.24$ for CIUC and $S = 0.25$ for CAUC on varved clay at the Amherst test site. Based on field vane tests and lab simple shear tests, they assign an empirical exponent $m = 1.0$ for these soils. Alternatively, CSSM provides the expression [16, 19]:

$$S_{nc} \sigma_{vo} = \left( \frac{M_c}{2} \right) \left( \frac{YSR}{2} \right)$$

(23)

Using the consolidation test results for YSR together with $M_c = 0.94$ and $\Lambda = 0.9$ provides the benchmark values of $s_{uc}$ at Hartford, as seen in Figure 13. The corresponding profile of $s_{uc}$ from the CPT interpretation is also presented and provides comparable values with depth that are consistent with the aforementioned analyses.

4 Additional applications

Additional case studies of the approach for obtaining $I_k$ in clays are given in related studies [31]. Moreover, the direct assignment of the operational value for $I_k$ from CPTu has application in the interpretation of the coefficient of consolidation ($c_v$) and permeability ($k$) of clays from piezo-dissipation tests [24, 32, 33, 34].

5 Conclusions

The interpretation of piezocone tests in soft to stiff clays can be systematically handled through two sets of analytical solutions: (a) NTH limit plasticity solution to evaluate the effective friction angle, $\phi'$; and (b) hybrid SCE-CSSM framework that provides the rigidity index ($I_k$), yield stress ratio (YSR), cone bearing factor ($N_kT$), and undrained shear strength ($s_u$) that corresponds to a triaxial compression mode. A case study is presented with SCPTu results obtained in an overconsolidated varved clay in downtown Hartford, Connecticut to illustrate the procedures. Supplemental lab tests including one dimensional oedometer tests and both drained and undrained triaxial shear tests provide validation on the CPTu procedures and interpretations.

References