Consolidation coefficient from cone penetration-based dissipation tests

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ABSTRACT: The Cone Penetration Test (CPT) apparatus is one of the most used instruments for carrying out field investigations of mechanical properties of soils. Several important parameters can be interpreted from the test. The amount information obtained from CPT is increasing as several sensors can be fitted into the instrument. Piezometer is one of them. During the CPT operation, excess pore pressure is generated as the instrument is pushed into soils of considerably low permeability such as clays. The generated pore pressure is then measured by a piezometer fitted close to the tip and around the shoulder of the piezocone. The dissipation of pore pressure in time can be registered by pausing the penetration process and recording the pore pressure in time. The horizontal coefficient of consolidation can then be determined from the pore pressure versus time readings. In this paper, selected methods that are suggested in literature for the evaluation of horizontal coefficient of consolidation from piezocone tests are studied. The methods studied are found to grossly disregard the consolidation in the vertical direction. In the paper, analytical solutions that consider consolidation in both the vertical and the radial direction are put forward.

Keywords: CPTU; horizontal consolidation coefficient; consolidation; consolidation in a cylindrical coordinate system

1. Introduction

The Cone Penetration Test (CPT) is one of the widely and popularly applied *in situ* testing methods in the Geotechnical Engineering field. Various mechanical properties of soils are evaluated from CPT measurements. A particular interest here is the piezocone which provides three independent measurements, namely the tip resistance, the sleeve resistance and the pore pressure. These measurements can be interpreted to find the stiffness, the strength and the flow parameters of the soil. Fig.1 shows important parts of the cone penetration test.

The test is performed by pushing the piezocone into the soil. As the instrument penetrates the soil, it causes changes in stresses and pore pressures in its vicinity due partly to shear induced at the sleeve-soil boundary and due partly to cavity expansion as the surrounding soil mass is pushed out. Generated excess pore pressure can subside very fast in highly permeable soils. However, in soils with low permeability, such as clays, it takes time to dissipate. The instrument can then be paused at a desired depth for monitoring the rate of dissipation. The data so obtained can be used for evaluation of the coefficient of consolidation of the material which is an important parameter in calculation of deformations due to consolidation.

2. Governing differential equation

The determination of the coefficient of consolidation from cone penetration tests can be considered as an inverse boundary value problem. That is, given the pore pressure and its dissipation in time, we would like to find the corresponding coefficient of consolidation, when applied together with the consolidation equations can give satisfactory approximation to the consolidation curves.

A cylindrical coordinate system appears to be a suitable choice when considering the consolidation process along the shaft. Whereas, a spherical coordinate system is a more suitable choice for writing down the differential equations that describe the consolidation process at and close to the tip of the CPT instrument.

The differential equation can then be written as

$$\frac{\partial u}{\partial t} = c_r \frac{1}{r^a}\frac{\partial}{\partial r}\left(r^a \frac{\partial u}{\partial r}\right) + bc_z \frac{\partial^2 u}{\partial z^2}, \quad (1)$$

in which a =1 and b=1 for cylindrical coordinate system (assuming $\partial u/\partial\theta$=0 and radial symmetry of material properties) and a = 2 and b = 0 for spherical coordinate system, c_r and c_z are coefficients of consolidation along the radial and the vertical direction respectively.

Note that c_r in the case of consolidation in spherical coordinate system is not truly radial in the sense of the cylindrical coordinate system.

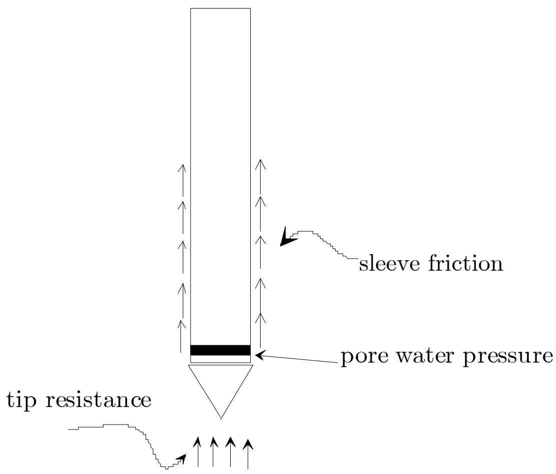

Figure 1. Simplified schematics of piezo cone penetration test apparatus

3. Horizontal consolidation coefficient from cone penetration tests-selected methods

The differential equations, Eq.1, can be solved analytically or numerically. Next, we will present some methods for the determination of horizontal coefficient of consolidation from Cone Penetration based dissipation tests and discuss some of their limitations.

3.1. Based on a predetermined time factor

The horizontal coefficient of consolidation may be written as

$$c_h = \frac{Tr_0^2}{t}, \quad (2)$$

where T is the time factor which depends on the degree of consolidation, r_0 is the filter radius. Often, in literature, the value of the time factor at the 50% degree of consolidation is considered representative.

Torstensson [1], considering a cylindrical and a spherical cavity expansion and solving the differential equations of consolidation, produced various plots of degree of consolidation versus time factor for varying rigidity index of the soil. The rigidity index of the soil is defined as the ratio of the secant stiffness of the soil to the undrained shear strength of the soil. The solutions by Torstensson indicated that consolidation happens faster in the case of the spherical cavity expansion than the cylindrical one.

For 50% degree of consolidation, the time factor values from Torstensson's [1] consolidation curves can be approximated by

$$T_{50} \approx 0.2 + 0.0012\, E/c_u \quad (3)$$

for spherical dissipation and

$$T_{50} \approx 0.7 + 0.0078\, E/c_u \quad (4)$$

for cylindrical dissipation, where E is the secant stiffness of the soil. Using the time to 50 % consolidation from the piezocone dissipation measurements together with Eq.3 or 4, one is then able to calculate the horizontal coefficient consolidation. Torstensson noted that the coefficient of consolidation evaluated from the theoretical curves he produced over predicted those measured in the field at least by a factor of two.

Senneset *et al.* [2] investigated the coefficient of consolidation from piezocone dissipation tests assuming an initial logarithmic pore pressure profile and describing the consolidation process using a one-dimensional consolidation equation. They proposed that the radial coefficient of consolidation can be estimated from the rate of pore pressure dissipation using

$$c_h = \lambda_c r_0^2 \left| \Delta\dot{u}/\Delta u_i \right|, \quad (5)$$

where r_0 is the filter radius, $\Delta\dot{u}$ is the rate of pore pressure dissipation and Δu_i is the initial excess pore pressure. The parameter λ_c is to be determined from plots which give the values of λ_c for a normalized excess pore pressure ($\Delta u/\Delta u_0$) for different values of the rigidy index.

The various $\Delta u/\Delta u_0 - \lambda_c$ curves produced by Senneset *et al.* [2] can be approximated by a power function as

$$\lambda_c = a\left(\Delta u/\Delta u_0\right)^n \quad (6)$$

in which,

$$a = 0.1208\left(G/c_u\right)^{0.529}; \; n = -2.32 - 0.009\, G/c_u.$$

According to Senneset *et al.* [2], the method gave best agreement when measured stresses are greater than the pre consolidation stress, i.e., for normally consolidated condition. The curves by Senneset *et al.* do not use the full differential equation in the cylindrical coordinate system. Whether the method gives a realistic value for the horizontal coefficient of consolidation remains to be investigated.

Teh and Holusby [3] presented the results of their numerical investigation of the cone penetration test in clay employing the "Strain Path Method". In their analyses, for a constant rigidity index, they found that the degree of dissipation versus time factor curves greatly varied depending on where along the CPT they were taken. They also found that the curves differ depending on the input rigidity index. However, almost a unique dissipation curve was found when the pore pressure dissipation curves were plotted against what they called a modified time factor defined as time factor divided by the square root of the rigidity index. From their calculations, they found that the modified time factor, T^*, for a 50 % degree of consolidation was 0.069 at the cone tip, 0.118 at the cone face, 0.245 at the cone shoulder, 1.11 5 radii above the cone shoulder and 2.14 at 10 radii above the cone shoulder. Accordingly, the horizontal coefficient of consolidation can be found as

$$c_h = \frac{T^* r_0^2}{t} = \frac{T\sqrt{I_r}\, r_0^2}{t}. \quad (7)$$

Depending on where the piezometer is situated along the cone penetration instrument and assuming the 50% degree of consolidation is representative, the value of T^* can be picked. The time to 50% degree of consolidation can be red from the measured degree of consolidation versus time curve to determine the horizontal degree of consolidation. In so doing, however, the consolidation in the vertical direction is not given due consideration.

3.2. From analytical solutions

For a homogeneous medium, the consolidation equation in a cylindrical coordinate system can be solved analytically. Assuming radial consolidation, the differential equation for consolidation (the same as also heat flow in radial direction in a cylindrical coordinate system) and further assuming an initial excess pore pressure profile generated by cavity expansion is described by

$$u_{r0} = 4c_u \ln\left(r_0 I_r^{1/3}/r\right), I_r = G/c_u, \quad (8)$$

Randolph and Wroth [4] found that the differential equation of consolidation in the radial direction in a cylindrical coordinate system is satisfied by the solution

$$u(r,t) = \sum_{i=1}^{\infty} c_i \begin{Bmatrix} -Y_0(\alpha_i r)J_0(\alpha_i r_p) \\ +Y_0(\alpha_i r_p)J_0(\alpha_i r)\exp(-c_r \alpha_i^2 t) \end{Bmatrix}, \quad (9)$$

where J_0 and Y_0 are Bessel functions of order zero of the first and the second kind, r_p is the plastified radius (defines the radial extent of the intially genearted porepressures), α_i define the eigenvalues of the bessel functions (which are found by solving the equation $J_0(\alpha r_p)Y_1(\alpha r_0) - Y_0(\alpha r_p)J_1(\alpha r_0) = 0$) the coefficients c_i area determined from:

$$c_i = \frac{\int_{r_0}^{r_p} r u_{r0}\left[Y_0(\alpha_i r_p)J_0(\alpha_i r) - Y_0(\alpha_i r)J_0(\alpha_i r_p)\right]\mathrm{d}r}{\int_{r_0}^{r_p} r\left[Y_0(\alpha_i r_p)J_0(\alpha_i r) - Y_0(\alpha_i r)J_0(\alpha_i r_p)\right]\mathrm{d}r}. \quad (10)$$

This solution can be implemented in Excel. The eigenvalues, α_i, can be iteratively found using the inbuilt Excel solver by imposing the condition that $J_0(\alpha r_p)Y_1(\alpha r_0) - Y_0(\alpha r_p)J_1(\alpha r_0) = 0$. This condition arises from the fact that the cone is an impermeable boundary. The values of α depend on the r_p and hence must be solved for every change in the *plastified* radius, r_p, but remain constants for a given *plastified* radius, r_p. Their values seem to increase exponentially with their indices, Fig.2. They also seem to increase almost linearly with the value of r_p/r_0. For the start, they can be approximated by an exponential equation of the form:

$$\alpha_i = \xi e^{\chi i}, i = 1,2,3,\ldots \quad (11)$$

Furthermore, the contribution from higher terms seems to vanish exponentially and only the first few (say 20) terms give adequate approximation to the solution.

Burns and Myne [5] extended the solution by considering that the pore pressure generated in the radial direction is due not only to cavity expansion but also shear- and that the pore pressure generated due to shear is given by

$$u_{s0} = \sigma'_{v0}\left(1 - \left(0.5\mathrm{OCR}\right)^{\lambda}\right)\frac{\left(r - r_s\right)}{r_0 - r_s}, \quad (12)$$

where OCR is overconsolidation ratio, $\acute{\sigma}_{v0}$ is the initial effective stress, λ is a parameter and r_s is the radius at which the change in pore pressure due to shear is assumed to be zero. Note that the pore pressure can be negative or positive depending on the value of the OCR. The higher the OCR the more dilative the soil is the more the tendency to create suction pressure due to imposed almost isochoric condition on the tendency to dilate during shear.

They further assumed the additive superposition between the pore pressure contributions from cavity expansion and from shear as

$$u(r,t) = u_v(r,t) + u_s(r,t), \quad (13)$$

where $u_v(r,t)$ is as described in Eq.9 and $u_s(r,t)$ is given as:

$$u_s(r,t) = \sum_{j=1}^{\infty} c_j \begin{Bmatrix} -Y_0(\beta_j r)J_0(\beta_j r_p) \\ +Y_0(\beta_j r_p)J_0(\beta_n r)\exp(-c_r\beta_j^2 t) \end{Bmatrix}, \quad (14)$$

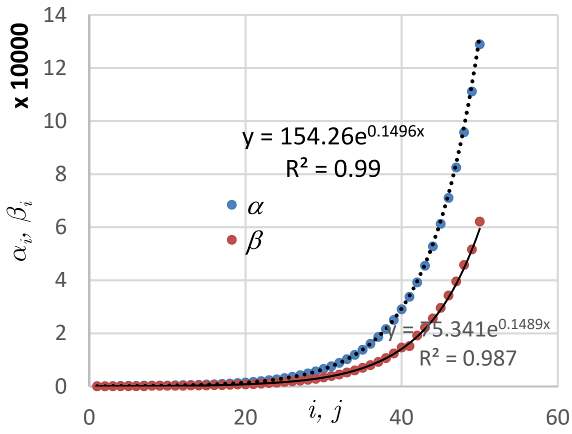

Figure 2. An example approximation of roots of $J_0(\alpha r_p)Y_1(\alpha r_0) - Y_0(\alpha r_p)J_1(\alpha r_0) = 0$.

where the coefficients c_j are given by:

$$c_j = \frac{\int_{r_0}^{r_s} r u_{s0}\left[Y_0(\beta_j r_s)J_0(\beta r) - Y_0(\beta_j r)J_0(\beta r_s)\right]\mathrm{d}r}{\int_{r_0}^{r_s} r\left[Y_0(\beta_j r_s)J_0(\beta_n r) - Y_0(\beta_j r)J_0(\beta_j r_s)\right]\mathrm{d}r} \quad (15)$$

Like the α_i, the β_j 's can be determined using the Excel solver imposing the boundary condition $J_0(\beta r_s)Y_1(\beta r_0) - Y_0(\beta r_s)J_1(\beta r_0) = 0$.

The horizontal degree of consolidation can then be determined by trial and error such that the measured dissipation curves are matched by the analytical equations.

3.3. An example interpretation of c_h based on the various methods

We will now employ the different methods presented previously for the determination of the radial coefficient of consolidation. A piezocone test was performed in a clay soil which has a rigidity index of 100. The pore pressure dissipation measurements were at three different depths and the piezometer was situated at the shoulder of the cone. The measured values are shown in Fig.3 and the horizontal coeffcient of consolidation evaluated after four different methods are shown in Fig. 4. In the evaluation:

- the average rate of pore pressure dissipation is assumed for the method by Senneset *et al.*
- $c_{u=}\ \alpha OCR^{0.5}\sigma'_{v0}$, where $\alpha = 0.2$ is considered and from shallow to deeper, OCR =2, 1.5 and 1.9 were assumed for fitting the dissipation curves using the analytical equations of Burns and Myne [5].

Calibration of Burns and Myne's equations is shown in Fig. 5.

It is seen that the analytical solution due to Burns and Myne [5] gave the least value for the horizontal coefficient of consolidation while Senneset *et al.*'s method gave the highest. On the other hand, the methods due to Torstensson [1] and Teh and Houlsby [3] gave almost the same result and both lie around the middle of those found from the analytical solutions of Burns and Myne [5] and that of Senneset *et al*. [2].

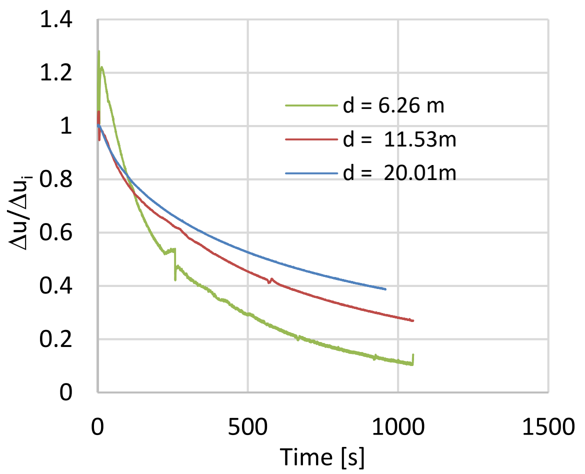

Figure 3. Example dissipation measurements at three different depths

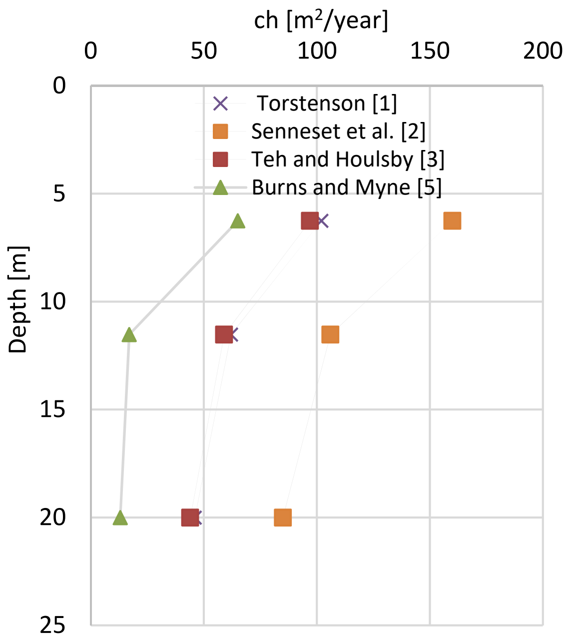

Figure 4. Horizontal coefficient of consolidation determined using three different methods

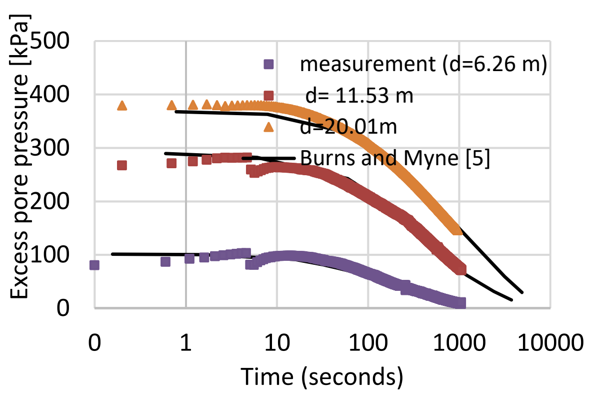

Figure 5. Back calculation of the excess pore pressure versus time curves by Burns and Myne's [5] solutions

3.4. Limitations

In general, the methods considered disregarded vertical consolidation. This may lead to a conservative estimate of the horizontal degree of consolidation, especially for shallow depths. The analytical solutions due to Burns and Myne [5] added pore pressure contributions from shear. In so doing, they accounted for pore pressures generated due to shear mobilizations at the cone soil interface. We must point out here that although it can be conceived that the total pore pressure generation is due to shear and cavity expansion induced by the penetration of the piezocone, using separate boundary conditions for each mechanism as it is done in Burns and Myne (2002) is not physical. The boundary conditions for the two mechanisms are not truly separable as the initial pore pressure generated due to each is there as a superimposed pore pressure from the start. The supposition that each pore pressure follows its own boundary conditions during the consolidation processes is also not physical.

4. Solutions considering both radial and vertical directions for consolidation

In this section, we would like to get back to the differential equation we presented in Eq.1 and attempt to provide some numerical and analytical solutions such that effect of vertical consolidation can be taken into account.

4.1. Numerical solutions using the Finite Difference Method

The differential equation, Eq.1, can be solved using numerical methods in its generality. For instance, in the cylindrical coordinate system, it can be approximated by a Finite Difference discretization as:

$$\begin{aligned}\frac{u_{ij}(t+\Delta t)-u_{ij}(t)}{\Delta t} &= c_r\frac{1}{r_{ij}}\frac{u_{ij+1}(t)-u_{ij-1}}{2\Delta r}\\ &+\frac{c_r}{\Delta r^2}\left\{u_{ij+1}(t)-2u_{ij}(t)+u_{ij-1}(t)\right\}\\ &+\frac{c_z}{\Delta z^2}\left\{u_{i+1j}(t)-2u_{ij}(t)+u_{i-1j}(t)\right\}\end{aligned} \qquad (16)$$

Further rearranging, the pore pressure at time $t+\Delta t$ at node i, j can be determined as:

$$\begin{aligned}u_{i,j}(t+\Delta t) &= u_{i,j}(t)+\frac{c_r\Delta t}{\Delta r}\frac{u_{i,j+1}(t)-u_{i,j-1}(t)}{2r_{ij}}\\ &+\frac{c_r\Delta t}{\Delta r^2}\left\{u_{i,j+1}(t)-2u_{i,j}(t)+u_{i,j-1}(t)\right\}\\ &+\frac{c_z\Delta t}{\Delta z^2}\left\{u_{i+1,j}(t)-2u_{ij}(t)+u_{i-1,j}(t)\right\}\end{aligned} \qquad (17)$$

in which i is a node counter in the z direction, j is a node counter in the radial direction, and Δt, Δr and Δz are increments in time, in vertical distance and in radial distance respectively, Fig. 6.

The scheme together with appropriate boundary conditions and steps handles a consolidation process in a cylindrical coordinate system and it is conditionally stable and begins to be unstable when $c_i\Delta t/\Delta x^2 \geq 0.5$.

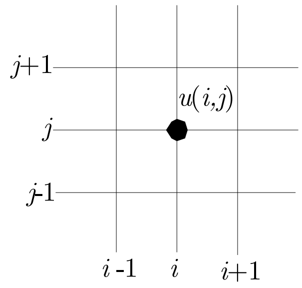

Figure 6. Illustration of the Finite Difference discretization

4.2. An extended analytical solution

For a homogeneous mass, analytical solutions may be derived assuming separation of variables in the form:

$$u(r,z,t) = u_r(r,t)\, f(z,t)\,, \tag{18}$$

where $u_r(r,t)$ can be defined by the solutions provided for radial consolidation in a cylindrical coordinate system, Eq. 9, (disregarding the additive decomposition of contributions from shear and cavity expansion), $f(z) = f(z, t = 0)$ desribes the initial excess pore pressure profile generated at the piezocone-soil interface in the vertical direction during the penetration of the CPT and the function $f(z,t)$ describes the evolution of $f(z)$ in time due to consolidation process and is given by

$$f(z,t) = \sum_{j=1}^{\infty} c_j \cos\left\{(2j-1)\pi \frac{H-z}{2H}\right\} \exp\left\{-c_z \frac{\pi^2 \pi^2 (2j-1)^2}{4H^2} t\right\} \tag{19}$$

in which assuming the boundary conditions: $\partial u/\partial z = 0$ at $z = H$ and $u = 0$ at $z = 0$, the coefficients c_j are defined as

$$c_j = \frac{2}{H} \int_0^H f(z) \cos\left\{(2j-1)\pi \frac{H-z}{2H}\right\} \mathrm{dz}\,, \tag{20}$$

which is completely specified once $f(z)$ that describes the profile of the initially generated pore pressure in the z-direction is specified. See Fig. 7 for illustration.

Note that Eq.18 is a combination of two well-known solutions. Eq.19 is the solution of 1D heat flow solved by Fourier and later adopted by Terzaghi for describing 1D consolidation process. In this way now, we have included effect of the consolidation in the vertical direction.

5. Conclusions

Coefficient of consolidation is an important parameter. Methods for determining the horizontal coefficient of consolidation from piezocone penetration tests have been investigated and their limitations were pointed out. It is seen that almost all the methods disregard the contribution due to consolidation in the vertical direction. This might lead to over estimation of the degree of consolidation in the horizontal/radial direction. In this paper, solutions that include the consolidation in the vertical direction are put forward. The full utilization of such considerations for the determination of coefficients of consolidation remains to be investigated.

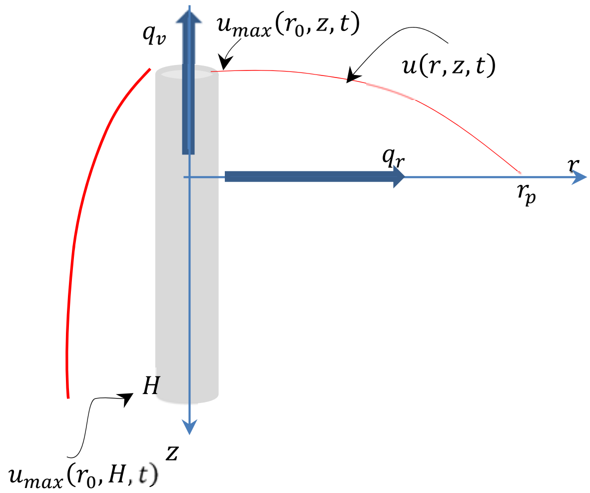

Figure 7. Sketch set up for initial pore pressure profile in the radial and in the vertical direction

Symbols

Variables

- r: Radial distance
- r_0: Pile radius
- r_p: Plastified radius
- u: pore pressure
- t: Time
- T: Time factor
- T^*: Modified time factor
- T_x: Time factor at a given degree of consolidation x.
- z: Vertical distance

Parameters/coefficients/constants

- c_r: Radial consolidation coefficient
- c_z: Vertical consolidation coefficient
- H: Pile height (length) or half the pile length in the case of two-way drainage
- c_i, c_j: coefficients that depend on the initial pore pressure profile
- α_i, β_i: Eigen values of the Bessel function, J_0 and Y_0

Functions

- J_0: Bessel function of the first kind and order zero
- Y_0: Bessel function of the second kind and order zero

Operators

- Δ: Finite increment
- $\frac{\partial}{\partial x}$: Partial differential wrt the variable x

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