# Determination of parameters for HS and SS model for Transdanubian clay

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**ABSTRACT:** In recent years, in the field of geotechnical design, software based on FEM has come to the front. Advanced computer programs make it possible to use advanced soil models besides the most current elastic-plastic Mohr-Coulomb model. By using these computer programs, nonlinear behavior of the soil can be described more realistic, even in the case of more complicated load events. Among the constitutive models incorporated in the commercial software the Hardening Soil Model (HS), Hardening Soil *model* with small-strain stiffness (HSsmall) and Soft Soil Model (SS) are the most promising ones. Observations and experience gained in tunnel construction, deep open excavation or preloaded embankment prove that with these soil models reality can be followed more accurately, especially in cases where unloading and reloading are present.
In order to produce the software input parameters, in case of more complicated soil model, more demanding laboratory tests are needed. In case of HS model one of the basic demands is to determine the power for stress-level dependency of stiffness (m), tangent stiffness for primary oedometer loading ( $E_{oed}^{ref}$ ) and unloading/reloading stiffness ( $E_{ur}^{ref}$ ). For SS model determination of the modified compression index (λ*) and modified swelling index (κ*) are esential.
The paper focuses on the behavior of transdanubian clay common in Hungary. Sampling, laboratory investigations and evaluation aimed to determine the input parameters for the HS and SS are described. Results based on a number of oedometric tests accomplished with unloading and reloading proved to be adequate for the computational purposes. The paper does not focus on the HSsmall model due to the difficulty of determination of the input parameters from oedometer test.

**Keywords:** Clay, Hardening Soil Model, Soft Soil Model

## 1. Introduction

Due to the increasing demands of construction projects, civil engineers must cope with the challenges of unloading/reloading behavior. The typical cases are deep excavations in an urban area or applying and removing preload to an embankment. Movements induced by excavations demand more accurate analysis of soil-structure interaction, especially in urban areas. In such cases, strains and displacements cannot be descried accurately by the conventional constitutive model: the elasto-plastic model with Mohr-Coulomb failure criteria. Excavations and reloading can be analyzed only by a constitutive model capable of describing nonlinear behavior and the hardening process [1]. Advanced geotechnical software like PLAXIS or MIDAS now make it possible to use more complex soil models. The two most promising new models are: the Hardening Soil Model (HS); and the Soft Soil Model (SS). Observations and experience gained in tunnel construction, deep excavations, and preloaded embankments show that these soil models describe field behavior more accurately. This is especially true where there are complex loading and unloading sequences in construction [2].

Geotechnicans using these computational tools often face the problem of not having adequate or accurate data for preliminary geotechnical estimations or recommendations for structural design. To overcome this difficulty, the laboratory at Széchenyi István University has performed approximately two hundred oedometer tests using unloading and reloading during the last few years. We have analyzed the test results and established correlations for the computational model parameters. The aim of this research was to determine reliable relationships to serve as a design aid during the preliminary phase of a project.

## 2. Investigated soil properties

The samples originated from different regions of Hungary. Liquid limit and plasticity index of the tested soils are plotted on the Casagrande chart (Fig.1).

The points are above the A-line and diverge from the A-line with increasing liquid limit values. The equation of the best-fit line is

$$I_p = 0{,}86 \cdot (w_L - 15{,}5) \quad (1)$$

The correlation coefficient is $r$ = 0.95. Based on the Casagrande chart, the tested samples were mainly medium and high plasticity clay but with some silt too.

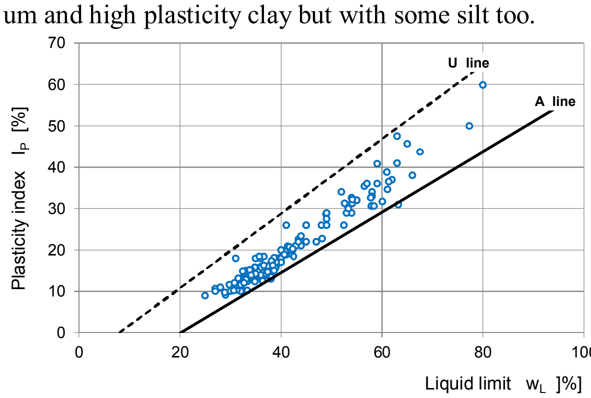


**Figure 1.** Correlation of $I_P$ - $w_L$

Table 1. summarizes the main parameters of the tests. The plasticity index varies between 9 % and 60 % with an average value of 21 %. The consistency index is be-

tween 0.2 and 1.8, with an average value of 1.0. The average oedometer modulus due to primary compression is 11 MPa, and the average elastic unloading/reloading modulus is 54 MPa. Note that the unloading/reloading is about five times the primary compression. This value fits into the recommendations of the PLAXIS manual which states that the quotient should be between three and five. The collected data have been evaluated statistically and correlations have been evaluated, and these results are presented below.

**Table 1.** Statistical parameters of the soil properties

| parameters | | min | max | average | deviation | median |
|---|---|---|---|---|---|---|
| $w_o$ | % | 12.14 | 34.47 | 22.10 | 3.92 | 22.78 |
| $e_o$ | - | 0.48 | 0.88 | 0.64 | 0.07 | 0.64 |
| $I_p$ | % | 9.00 | 59.90 | 20.70 | 9.69 | 18.0 |
| $I_c$ | - | 0.18 | 1.80 | 0.96 | 0.26 | 0.94 |
| $z$ | MPa | 3.20 | 74.80 | 25.10 | 16.27 | 21.30 |
| $E_{oed}$ | MPa | 3.80 | 31.70 | 10.90 | 4.69 | 10.20 |
| $E_{ur}$ | MPa | 10.30 | 157.5 | 53.70 | 27.92 | 49.69 |
| $A$ | - | 0.007 | 0.114 | 0.031 | 0.02 | 0.03 |
| $B$ | - | 0.241 | 0.966 | 0.572 | 0.15 | 0.56 |
| $C_c$ | - | 0.032 | 0.191 | 0.106 | 0.03 | 0.105 |
| $C_s$ | - | 0.004 | 0.037 | 0.018 | 0.01 | 0.019 |
| $\lambda^*$ | - | 0.009 | 0.044 | 0.028 | 0.007 | 0.028 |
| $\kappa^*$ | - | 0.001 | 0.009 | 0.005 | 0.002 | 0.005 |

## 3. Parameters of Hardening Soil Model

The Hardening Soil model (HS) is an advanced model for simulating the behavior of different types of soil, both soft and hard. A basic feature of the present HS model is the stress dependency of soil stiffness. The HS model uses moduli both from oedometer tests and from triaxial tests. Oedometer test results demonstrate the dependence of stiffness on confining stress. The increase of the oedometric modulus depends on the mean hardening stress. This is described by Eq.(2)

$$E_{oed} = E_{oed}^{ref}\left\{\frac{c\cdot cos\varphi - \frac{\sigma_3'}{K_0^{NC}}\cdot sin\varphi}{c\cdot cos\varphi + p^{ref}\cdot sin\varphi}\right\}^m \quad (2)$$

where the $E_{oed}$ is a tangent stiffness modulus obtained from an oedometer test. $E_{oed}^{ref}$ is a tangent stiffness at a vertical stress of $-\sigma_1' = \frac{-\sigma_3'}{K_0^{NC}} = p^{ref}$. The reference pressure, $p^{ref}$ is typically 100 kPa. The amount of stress dependency is given by the power m.

Under triaxial conditions, the parameter $E_{50}$ is the confining stress-dependent stiffness modulus for primary loading and is given by Eq.(3)

$$E_{50} = E_{50}^{ref}\left\{\frac{c\cdot cos\varphi - \sigma_3'\cdot sin\varphi}{c\cdot cos\varphi + p^{ref}\cdot sin\varphi}\right\}^m \quad (3)$$

where $E_{50}^{ref}$ is a reference stiffness modulus corresponding to the reference confining pressure $p^{ref}$.

For unloading and reloading stress paths, another stress-dependent stiffness modulus is used:

$$E_{ur} = E_{ur}^{ref}\left\{\frac{c\cdot cos\varphi - \sigma_3'\cdot sin\varphi}{c\cdot cos\varphi + p^{ref}\cdot sin\varphi}\right\}^m \quad (4)$$

where $E_{ur}^{ref}$ is the reference Young's modulus for unloading and reloading, corresponding to the reference pressure $p^{ref}$.

Application of the HS model requires the parameters $m$, $E_{oed}^{ref}$ modulus and $E_{ur}^{ref}$ modulus [3].

In order to determine the input parameters, the stress-strain behavior as a power curve as suggested by Janbu [4] was applied as indicated in Eq.(5)

$$\varepsilon_z = A\left\{\frac{\sigma_z}{p}\right\}^B \quad (5)$$

where $\varepsilon_z$ is the vertical specific strain, $\sigma_z$ is the vertical stress, A is the scale factor relating stress and strain, $B$ is the power factor relating stress and strain, and p equals 100 kPa. The same shape as presented in the HS model was yielded by this formula. Taking the derivative of Eq.(5), the oedometric tangent modulus was determined as a function of $\sigma_z$ and is expressed in Eq.(6)

$$E_s = \frac{p}{A\cdot B}\left\{\frac{\sigma_z}{p}\right\}^{1-B} \quad (6)$$

### 3.1. Parameter analysis for HS model

Fig. 2. shows the relation between $A$ and $B$. According to Smoltczyk [5], parameter $B$ should depend on the type of soil and parameter $A$ should depend on the state of the soils. Considering our test results, there is a very strong relation between A and B parameters. Based on all the tests, the correlation can be descried by

$$A = 0.012 \cdot B^{-1,36} \quad (7)$$

The correlation coefficient is $r = 0.84$, implying good correlation. Based on Fig. 2, parameter $B$ slightly changes on the plasticity lines. The compression curve can be descried by these $A$ and $B$ parameters.

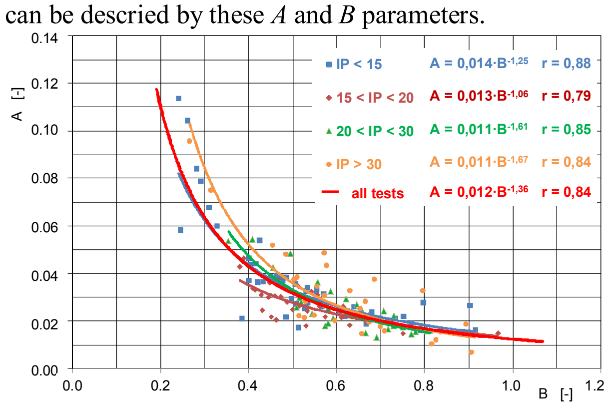


**Figure 2.** A and B parameters from oedometer test results

Based on previous research, correlations between parameter $A$ or $B$ and other simple soil properties were further investigated but, unfortunately, no relations were found. Fig. 3 shows the relationship between plasticity index and parameter $B$. One can see that parameter $B$ varies between 0.4 and 0.8 but slightly increases with plasticity index ($I_P$). The average value of parameter $B$≈0.5-0.55.

Fig. 4. shows the relationship between consistency index and parameter $A$. It seems that value of parameter $A$ slightly decreases with consistensy index ($I_C$), but values of parameter $A$≈0.015-0.04. The average value of parameter $A$≈0.03.

Fig. 5. shows the relationship between primary loading modulus and depth of the sample. It can be seen that modulus ineases with depth as usual. The equations are given for different plasticity, but for all the tests Eq.(8) describes the relation in MPa.

$E_{oed} = 0.25 \cdot z + 4.60$ (8)

The slopes of the regression lines vary slightly with plasticity.

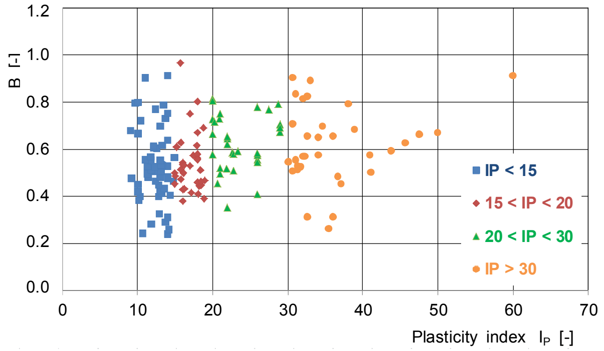


**Figure 3.** Relationship between $I_P$ and B

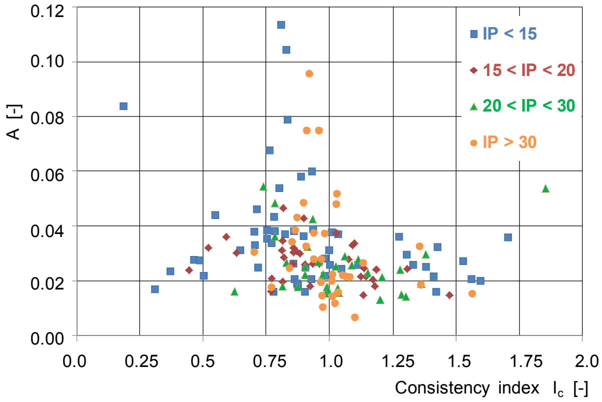


**Figure 4.** Relationship between $I_c$ and A

Similar equations were found for Hungarian Loess by Varga [6], and for Frankfurt Clay by Katzenbach [7].

Silty specimens (Ip<15) showed only a slight dependence on depth to their maximum at 40m.

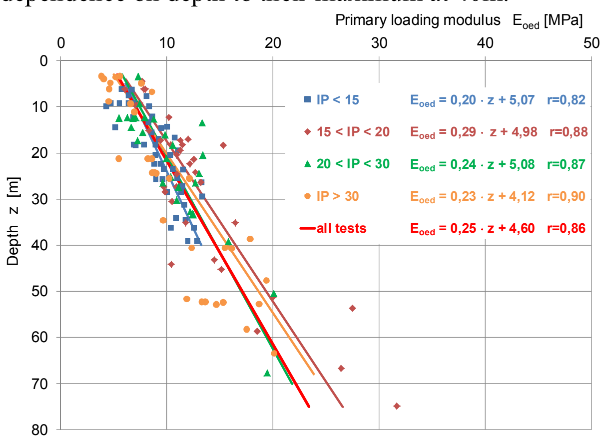


**Figure 5.** Relation between depth and $E_{oed}$

Note that dependency on depth due to the effect of mechanical state (effective stress and the volumetric deformation from origin). For normally consolidated soils it usually corresponds to the depth, therefore it is a suitable correlation and in practice we use *E=f(z)* type equations due to their simplicity. From a more scientific point of view, the stress-strain relation should be modeled by a nonlinear relation and the soil characterisation should be independent from the confining stress (ie depth) such as the modified compression index $\lambda^*$ in theSoft Soil model.

The relation between oedometer moduli due to primary compression andmodulidue to unloading/reloading is shown on Fig. 6. It can be established that, as plasticity increases, modulus ratio decreases, but the differences are not great.

The unloading/reloading modulus is ~4-5 times higher than the primary modulus with a correlation coefficient of r = 0.85. Considering all the test results, the ratio between primary compression and unloading/reloading compression can be expressed by

$E_{ur} = 4.95 \cdot E_{oed}$ (9)

The result agrees with the recommendation of PLAXIS manual that is 3 to 5. Note that a ratio of 5 was also back-calculated from field measuremnts on clayey soils in the southern part of Hungary.

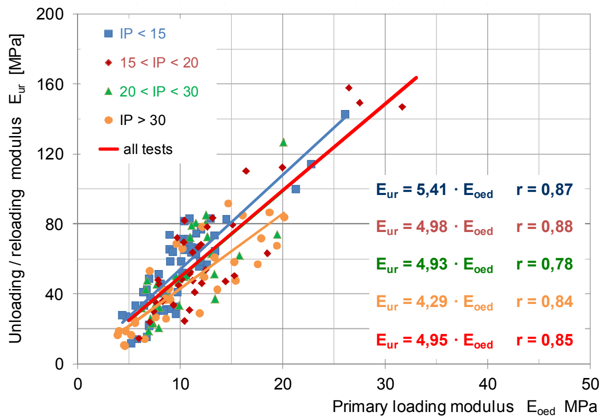


**Figure 6.** Relationship between $E_{oed}$ and $E_{ur}$

## 4. Parameters of Soft Soil Model

The Soft Soil model is suitable for materials that exhibit high degrees of compressibility, such as normally consolidated clays, clayey silts and peat. Based on this, volumetric hardening is the dominant feature that should be considered in constitutive modelling. Of course, a shear strength criterion is also needed for these geomaterials and a Mohr Coulomb yield surface is considered for this purpose [3].

The volumetric mechanism that captures the compressibility of the material is simulated by an elliptical cap that is very similar to the Modified Cam Clay model. Some features of the Soft Soil (SS) model are: a) stress-dependent stiffness (logarithmic compression behaviour); b) distinction between primary loading and unloading-reloading; c) memory for pre-consolidation stress; d) failure behaviour according to the Mohr-Coulomb criterion [3].

In the SS model, a logarithmic relation between the volumetric strain $\varepsilon^v$, and the mean effective stress *p*‘, is assumed. It can be formulated as Eq.(10)

$$\varepsilon_v - \varepsilon_v^0 = -\lambda^* \cdot ln\left\{\frac{p'+c \cdot cot\varphi}{p^0+c \cdot cot\varphi}\right\} \quad (10)$$

The minimum value of p' is set equal to a unit stress. The parameter $\lambda^*$ is the modified compression index, which determines the compressibility of the material in primary loading. During isotropic unloading and reloading a different path (line) is followed, which can be expressed as

$$\varepsilon_v^e - \varepsilon_v^{e0} = -\kappa^* \cdot ln\left\{\frac{p'+c \cdot cot\varphi}{p^0+c \cdot cot\varphi}\right\} \quad (11)$$

The parameter $\kappa^*$ is the modified swelling index, which determines the compressibility of the material in unloading and subsequent reloading. The relationship between the unloading/reloading elastic modulus, Poisson's ratio and modified swelling index is characterized by the following relation

$$\frac{E_{ur}}{3(1-2\nu_{ur})} = K_{ur} = \frac{p'+c{\cdot}cot\varphi}{\kappa^*} \quad (12)$$

where $\nu_{ur}$ is the Poisson's ratio for unloading-reloading [3].

This model is based on input parameters from oedometer tests. The stiffness parameter is not strictly the oedometer modulus, but the compression modulus. This modulus relates the volumetric strain to the log of mean effective stress. If volumetric strain is plotted against ln $p'$ (mean effective stress), parameters $\lambda^*$ and $\kappa^*$ can be obtained from the graph, as it is shown in Fig. 7.[3].

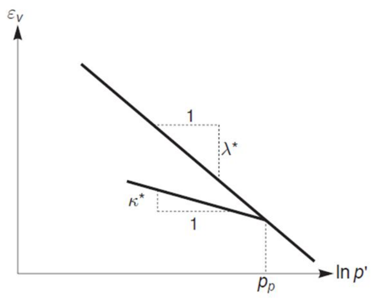


**Figure 7.** Logarithmic relation between volumetric strain and mean stress [3]

International literature offers a wide range of correlation between $C_c$ and $C_s$ index. Based on our test results for clayey soils, the average value of the compression index is $C_c$≈0.1, which corresponds to recommendations in Hungarian literature [8].

## 4.1. Parameter analysis for SS model

The best relationship occured between the compression index and the original water content as shown in Fig. 8. Based on all test results, the relationship between water content and compression index can be described by

$$C_C = 0.007 - (w_0 - 5.9) \quad (13)$$

with a correlation coefficient r = 0.85.

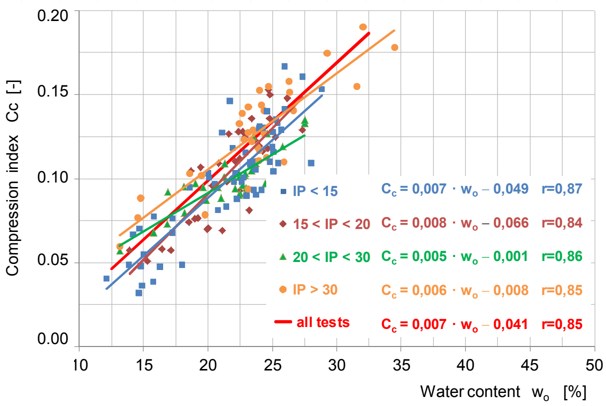


**Figure 8.** Relationship between $C_C$ and $w_0$

Fig. 9. shows the correlation between compression index and void ratio. The correlation coefficient is weaker than for water content. The best-fit correlation can be described by

$$C_C = 0.34 - (e_0 - 0.32) \quad (14)$$

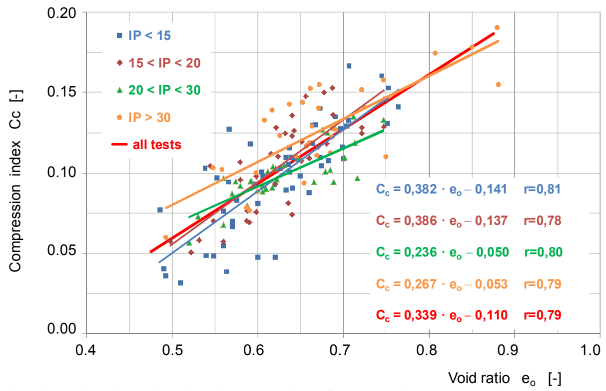


**Figure 9.** Relationship between $C_c$ and $e_0$

In related literature, several recommendations can be found for the relationship between compression index and water content or void ratio [9]. Some of them are collected in Table 2 according to Das [10]. Eq. (13) and (14) are in agreement with the equations given in Table 2.

**Table 2.** Recommendations for compression index

| | | |
|---|---|---|
| Skempton [1944] | moulded clay | $C_c = 0.007 \cdot (w_L - 7)$ |
| | Chicago-clay | $C_c = 0.013 \cdot w_o$ |
| Hough [1957] | inorganic cohesion soils | $C_c = 0.3 \cdot (e_o - 0.27)$ |
| Nishida (1956) | clay | $C_c = 1.15 \cdot (e_o - 0.27)$ |
| | low plasticity soil | $C_c = 0.75 \cdot (e_o - 0.50)$ |

Fig. 10. shows the correlation between modified compression index ($\lambda^*$) and water content ($w_0$). The relationship is weaker than it is shown on Fig. 8. and the different plasticity lines show less consisten behavior. For all data, the relationship can be described by

$$\lambda^* = 0.0015 - (w_0 - 3.3) \quad (14)$$

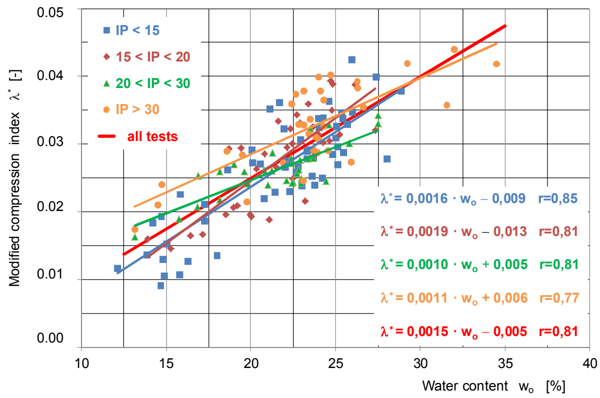


**Figure 10.** Relationship between $\lambda^*$ and $w_0$

Fig. 11. shows the strong relationship between swelling index (Cs) and compression index (Cc) with the compression index 6 times higher than swelling index. This is in good agreement with the international literature. The swelling index can be expressed by

$$C_s = C_c \cdot 0.175 = C_c/5.7 \quad (15)$$

Based on the graph, correlations for all types of soils are very similar. The correlation coefficient is r=0.84.

Relationship between $\lambda^*$ and $\kappa^*$ was examined on Fig. 12. The ratios are quite similar to those graphed on Fig. 11., but the correlation coefficient is a bit smaller. Based on all the data, the ratio between compression index and swelling index can be derived from

$$\frac{\lambda^*}{\kappa^*} = 5.7 \quad (16)$$

The ratio agrees with the recommendation from the PLAXIS manual of 2.5 to 7 [3]. Based on some back analyses, it is suggested that the SS advanved constitutive model is suitable for designing embankment foundations on soft soil.

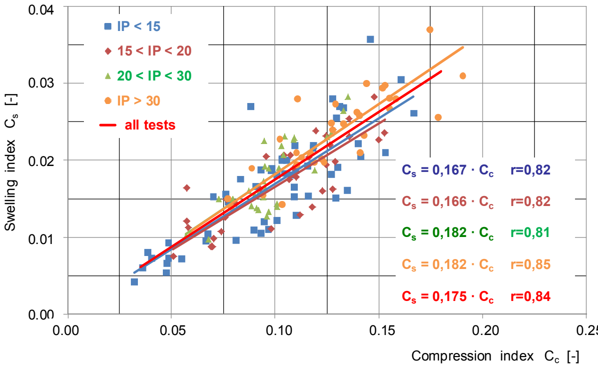


**Figure 11.** Relationship between $C_c$ and $C_s$

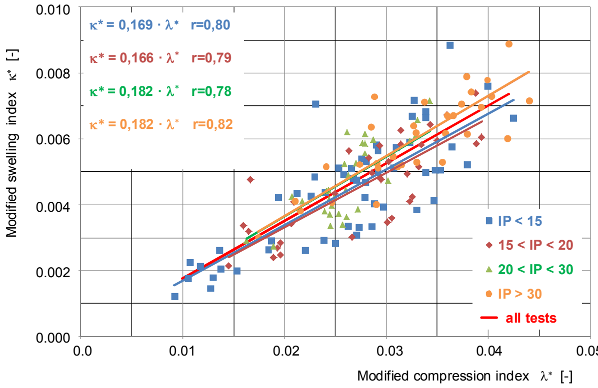


**Figure 12.** Relationship between $\lambda^*$ and $\kappa^*$

Fig. 13. shows an example for the application of SS model using PLAXIS 3D. An embankment with a height of h=5.5m was constructed in central Hungary. The top 3m of the subsoil was peat, resting on 6m of soft clay. To reduce settlement, dynamic replacement was executed. During contruction, systematic monitoring was carried out. After the execution of the highway section, a back-analysis was performed using PLAXIS 3D and MC and SS material models.

Only index parameters of the subsoil were tested in the laboratory. To determine the input parameters for the SS constitutive model, the above mentioned equations were applied. Result of the calculation with conventional method, PLAXIS 3D modeling using MC and SS material model and the result of the monitoring can be seen on Fig. 13. Results from modeling with SS model agreed very well with the results of the monitoring.

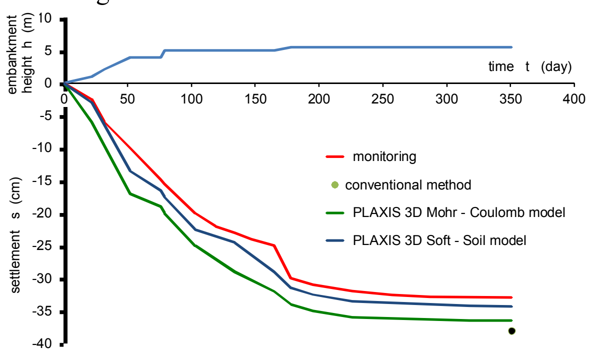


**Figure 13.** Settlement calculations and monitoring result

## 5. Conclusions

In order to produce the software input parameters for advanced soil models, more demanding laboratory tests are needed. The basic demands are to determine the power for stress-level dependency of stiffness ($m$), tangent stiffness for primary oedometer loading ($E_{oed}^{ref}$) and unloading/reloading stiffness ($E_{ur}^{ref}$) when using the Hardening Model. For the Soft Soil Model,the modified compression index ($\lambda^*$) and modified swelling index ($\kappa^*$) are needed. This paper focused on the input parameters of Hardening Soil Model (HS) and Soft Soil Model (SS) for Transdanubian Clay. More than 150 oedometer test were analysed focusing on the determination of these parameters. Based on the data analysis, Table 3. summarizes the equations established for Transdanubian clay.

**Table 3.** Determined correlations

| Hardening Soil Model | Sof Soil Model |
|---|---|
| $A = 0.012 \cdot B^{-1{,}36}$ | $C_s = 0.007 - (w_0 - 5.9)$ |
| $E_{oed} = 0.25 \cdot z + 4.60$ | $C_s = 0.34 - (e_0 - 0.32)$ |
| $E_{ur} = 4.95 \cdot E_{oed}$ | $\lambda^* = 0.0015 - (w_0 - 3.3)$ |
| | $\lambda^*/\kappa^* = 5.7$ |

These parameters agree with the range of recommended values from the PLAXIS manual, but give a more detailed description, especially for Transdanubian clay. These correlations can be applied by geotechnical engineers in cases where only index properties are available or when preliminary evaluations are performed. They also add confidence to the analyst when performing computations using PLAXIS or other sophisticated programs.

## 6. References


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