

# Prediction model for shear wave velocity of gravel

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**ABSTRACT:** Several studies have established empirical correlations between shear wave velocity ( $V_s$ ) and standard penetration test blowcount ( $N$ ) for engineering use. However, these empirical correlations cannot be applied to gravelly soil since the obtained  $N$  is usually not meaningful for gravel. Therefore, an empirical correlation of  $V_s$  for gravel is developed in this study using the Engineering Geological Database for the Taiwan Strong Motion Instrumentation Program. The  $V_s$  prediction model accounts for the Influences of confining stress, fines content, and plastic index in addition to the gravel contents. The model successfully applies to different regions in Taiwan and, thus, is potentially to be used for the other countries.

**Keywords:** shear wave velocity; gravel; prediction model.

## 1. Introduction

A key property required to effectively estimate the seismic response of a site is small-strain shear modulus  $G_0$ , which is often computed by measuring shear wave velocity ( $V_s$ ) and mass density ( $\rho$ ) as follows:

$$G_0 = \rho V_s^2. \quad (1)$$

The importance of  $G_0$  has been widely recognized in site response analysis and ground motion prediction. Such information ( $G_0$  or  $V_s$ ) is usually lacking during field exploration, but standard penetration test blow count ( $N$ ) is typically available. Therefore, several studies have established empirical correlations between  $N$  and  $V_s$  for engineering use. However, these empirical correlations cannot be applied to gravelly soil since the obtained SPT- $N$  is usually meaningless for gravel.

An empirical correlation for gravelly soils is developed in this study using the Engineering Geological Database for the Taiwan Strong Motion Instrumentation Program. Influences of confining stress, fines content (FC), plasticity index (PI) on  $V_s$  are first evaluated through the developed correlations with these parameters for a wide range of soil types except for gravel. A  $V_s$  prediction model for gravel is then proposed through the residual analysis to account for the effect of gravel content (GC). The model successfully applies to different regions in Taiwan that includes various characteristics of soil deposits and, thus, is potentially used for the other countries.

## 2. Correlation Form of $V_s$

### 2.1. Fundamental Functional Form of $G_0$

The most common functional form of the relations of  $G_0$  in the literature [1] is proposed as follows:

$$G_0 = A \cdot F(e)(\sigma_0')^n, \quad (2)$$

where  $F(e)$  is the function of void ratio, and constants  $A$  and  $n$  are determined by statistical regression of experimental results.

A summary by [2] indicates that  $n$  is mostly 0.5 for sand and various types of clays at small strain level (i.e.,  $n = 0.25$  for  $V_s$  according to Eq. (2) based on 11 proposed models. Kishida and Tsai [3] also reported that no clear dependency of  $n$  on soil type exists based on the analysis of field data and many previous studies. Therefore, the soil type is not a major factor that reflects the influence of confining stress on  $V_s$ .

### 2.2. Other Factors that Influence Functional Form

Hardin [4] discussed the model form of  $G_0$  for normally consolidated (NC) and overconsolidated (OC) clays and suggested the following:

$$G_0 = A \cdot F(e)OCR^k(\sigma_0')^n \quad (3)$$

where  $k$  can be approximated as

$$k \approx PI/160 \quad (4)$$

The equation implies the following. First, for a non-plastic or low PI soil (e.g., sand or silt),  $G_0$  is independent of OCR. Second, given  $OCR = 1$  (NC clay), Eq. (3) yields Eq. (2) and exponent  $n$  is a constant regardless of PI. Third, the impact of confining stress on  $G_0$  is uncoupled with OCR. Viggiani and Atkinson [5] also reported that the impact of confining stress on  $G_0$  is uncoupled with OCR as

$$G_0 = B(\sigma_0')^n OCR^m \quad (5)$$

However,  $B$ ,  $n$ , and  $m$  are dependent on PI as revealed by laboratory test results. Kawaguchi and Tanaka [6] proposed a semi-theoretical form of  $G_0$  as follows:

$$G_0 = 20000 \cdot w_L^{-0.8} \cdot \left(\frac{2}{3}OCR\right)^{0.2} \cdot \left(\frac{1 + OCR^{0.5}}{3}\right)^{0.6} \cdot (\sigma_0')^{0.8} \quad (6)$$

where  $w_L$  is the liquid limit, which is similar to the role of PI in Eq. (4). Eq. (6) indicates that the effect of OCR and PI (or  $w_L$ ) on  $G_o$  is uncoupled unlike that in Eq. (3), where the influence of OCR and PI on  $G_o$  is coupled.

In addition to the OCR and PI, several studies found that FC can influence the measurement of  $G_o$  [7-9]. High FCs result in low  $G_o$ . Based on the laboratory test of specimen with non-plastic fines up to 25%, Wichtmann et al. [7] proposed a correct factor  $fr(FC)$  to  $G_o$  with FC other than zero. The effect of FC is also accounted for when estimating liquefaction resistance based on  $V_s$  [10].

### 2.3. Proposed model form

Based on the review of previous studies, the small-strain properties are dependent on  $\sigma'_o$ , PI, OCR, and FC. The influence of these parameters can be coupled or decoupled. For simplicity, one possible regression models of  $G_o$  are proposed as follows:

$$\ln(V_s) = a_0 + a_1 \ln(\sigma'_o) + a_2 \ln(FC) + a_3 \ln(PI) + a_4 \ln(OCR), \quad (7)$$

Note that the effect of PI and OCR on  $G_o$  is uncoupled in Model 1. Determining which model is superior remains unknown because the coupling effect between PI and OCR is still under debate as discussed earlier.

### 3. Database for regression analysis

The data provided in EGDT include stratum description, results of soil physical property tests (such as grain size distribution, uniformity coefficient, coefficient of gradation, void ratio, water content, specific gravity, unit weight, liquid limit, and PI), soil classification, P-wave and S-wave velocities, and SPT-N values. EGDT provides sufficient information for the regression analysis. However, the required model parameter OCR is unavailable in EGDT. Therefore, OCR is approximately estimated by Eq. (6) in this study. Effective vertical stresses are calculated with the given depth, unit weight, and water table elevation. Groundwater elevation is occasionally not recorded for some borings. In such cases, the P-wave velocity profile is utilized to identify the approximate elevation of the groundwater table. An abrupt transition from P-wave velocity lower than 500 m/s to higher than 1500 m/s is typically apparent in boring logs, clearly indicating the position of the groundwater table.

In EGDT, P- and S-wave velocities are measured with a suspension PS logger system. Velocity measurement is generally performed every 0.5 m, except for several drillings in the first and second years, in which velocity is measured every 1 m.

The boring ID is similar to the codes of the TSMIP stations that were assigned according to the abbreviations of the different regions of Taiwan Island, which are TAP, TCU, CHY, KAU, TTN, HWA, and ILA, as shown in Figure 1. Notably, these regions are not categorized by its geological unit but simply based on the province. The data from TAP, TCU, CHY, HWA, and ILA are used for regression analysis, and the remaining data (KAU and TTN in the southern region of Taiwan) are employed to verify the model. SPT blow counts that exceed approximately 50 correspond to a refusal condition.

Therefore, we excluded the data of N larger than 50 in the database. Furthermore, for non-plastic soil, PI is set as a unity; for soil without FC, FC is also set as a unity. A total of 3,684 data sets from 334 sites that include  $V_s$ , N,  $\sigma'_o$ , FC, PI, and estimated OCR are used for the regression analysis. The distribution of data is shown in Figure 2. The  $V_s$  mostly distribute between 150–400 m/s with mean of 258 m/s; the  $\sigma'_o$  is mostly less than 300 kPa with mean of 167 kPa; the OCR is mostly less than 3 with mean of 1.6; the PI is mostly less than 5 with mean of 2.6; the FC evenly distributes between 0% and 100% with mean of 40.7%. The estimated OCR is high near the ground surface and decreases with the depth, which is consistent with the typical trend of field observation [11]. The evenly distributed FC indicates uniformly distributed soil type in the database, which is excellent for developing a unified model. The database consists of clay (29%), silt (21%), sand (48%), and gravel (3%).

Figure 3 shows the  $V_s$  against the model parameters.  $V_s$  is approximately linear against the model parameters in log-log space, which indicates that modeling  $V_s$  by Eqs. (7)–(8) is sufficient prior to the performance of regression analysis.

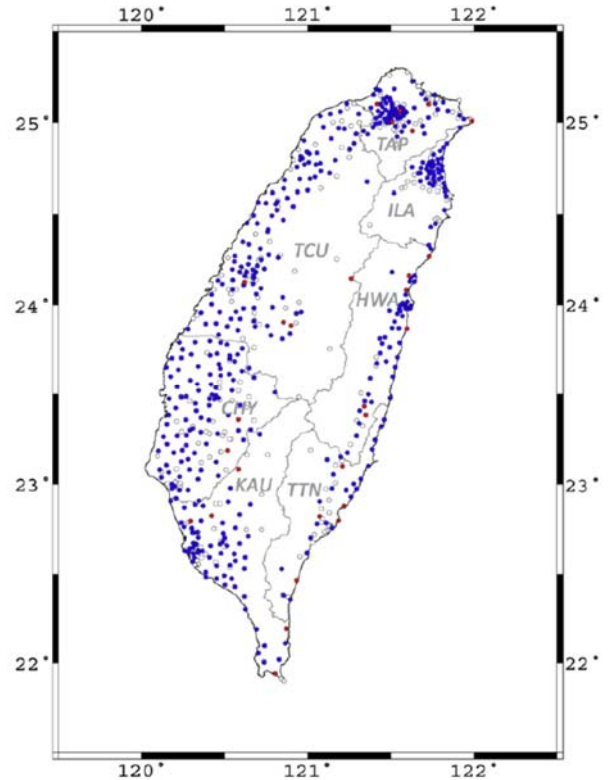


Figure 1. EGDT boring locations

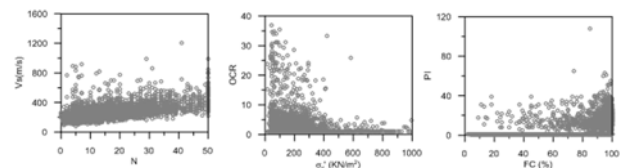


Figure 2. Distribution of data sets

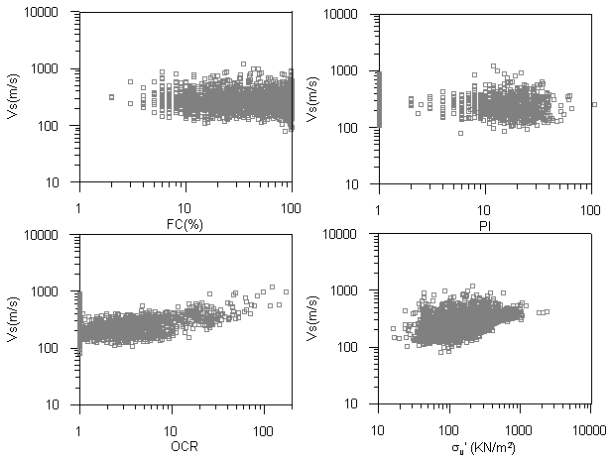


Figure 3. Vs against model parameters in log–log space

## 4. Regression analysis results

### 4.1.1. Model of non-gravel soil

Table 1 summarizes the regression result of Model 1 and Figure 4 shows the residuals of Vs of Model 1. Overall, the models do not demonstrate the bias to all the model variables, indicating the adequacy of the model form. The only bias is exhibited at the confining stress (or the shallow depth) where the mean residual is positive (i.e., underestimated). This bias is due to the inherent limitations of the model that describes the Vs as a constant exponential of  $\sigma'_v$ . The predicted Vs rapidly decreases and reaches zero as they approach the ground surface, while the measured Vs still presents certain values.

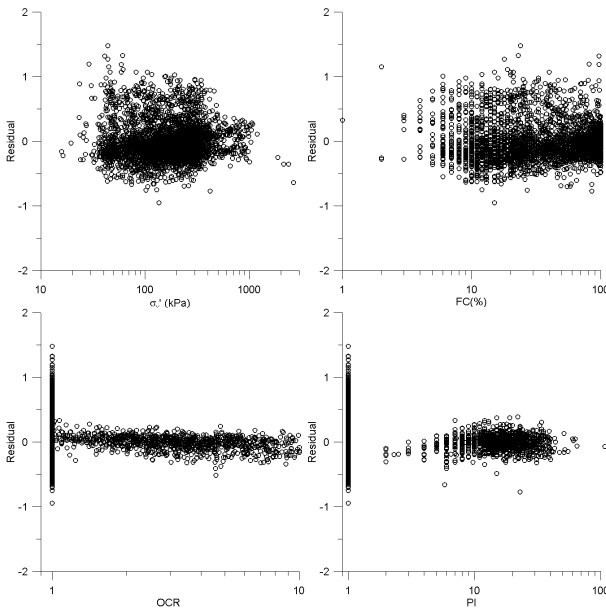


Figure 4. Residual of Vs prediction based on Eq.

Table 1. Regression result of Model 1

| Intercept | Exponent         |       |       |      | R <sup>2</sup> | lnσ  |
|-----------|------------------|-------|-------|------|----------------|------|
|           | σ <sub>v</sub> ' | FC    | PI    | OCR  |                |      |
| 4.59      | 0.26             | -0.08 | -0.18 | 0.32 | 0.42           | 0.28 |

### 4.1.2. Model of gravel soil

The Vs model (Eq. (7)) developed based on the database of TAP, TCU, CHY, HWA, and ILA is checked by the residual of prediction against the gravel contents. The database still includes 5% gravel (i.e. grain size > sieve #4) even the data of SPT-N > 50 is not used in the model development. As shown in Figure 5, the residual shows a positive trend against gravel contents. The positive trend of residuals indicates that the Vs of gravel is underestimated as the gravel content increases. The bias can be approximated as shown in Figure 5. Therefore, the Vs model of gravel is modified from Eq. (7) as

$$\ln(V_s) = 4.95 + 0.26\ln(\sigma'_v) + 0.08\ln(FC) + 0.18\ln(PI) + 0.32\ln(OCR) + 0.1\ln(GC) \quad (8)$$

Chang et al. [12] measured the Vs of sandy soil with different GCs by bender element tests. Similarly, the positive trend of Vs dependent on GC is found in their study. Figure 6 compares the GC correction, which is defined as  $V_s/V_s(GC=0)$ , calculated by Eq.(8) and Chang et al. [12]. The result of this study indicates a greater influence of GC on Vs (i.e. a higher GC correction) than that observed in [12]. The difference may be due to that [12] is only based on sandy soils with a constant Vs (i.e. one specific condition) while this study represents a general trend for different soil types with a wild range of Vs.

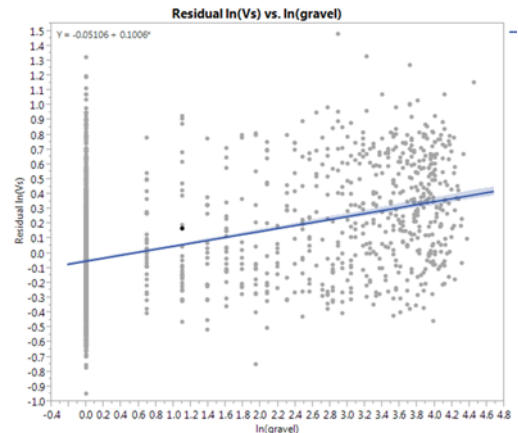


Figure 5. Residual of Vs prediction against GC in TAP, TCU, CHY, KAU, TTN, HWA, and ILA

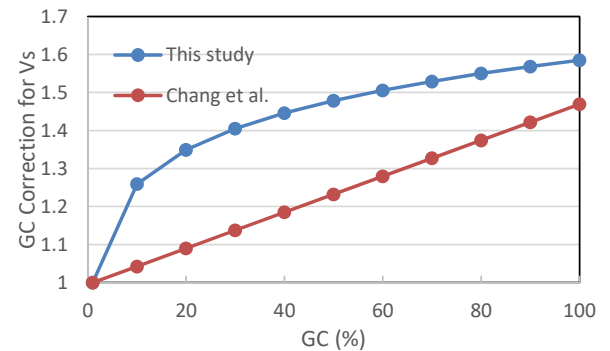
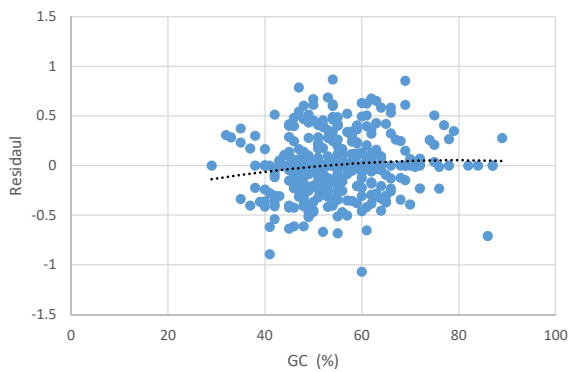


Figure 6. Comparison of GC correction in this study and Chang et al. [12]



**Figure 7.** Residual of Gravel Vs prediction against GC in all regions

This model is further validated by the Vs of gravel in all regions (TAP, TCU, CHY, KAU, TTN, HWA, and ILA, total 384 datasets). The residual of the prediction is shown in Figure 7. It can be seen that there is no clear bias against gravel contents, indicating a success of the model prediction for Vs of gravel in different regions. It is also noted that for  $\ln(GC) > 3.2$  (i.e.  $GC > 25\%$  or gravel), the standard deviation  $\ln\sigma$  is 0.33, which is higher than 0.26 for the prediction model of the other types of soils (Eq. (7)).

## 5. Conclusions

Numerous relations between N and Vs have been proposed for practical purposes in earthquake engineering. These empirical correlations, however, are not applicable for gravel because N of gravel is not meaningful due to refusal. Therefore, an empirical correlation model for gravel was developed without depending on N in this study. EGDT with 3684 data sets was utilized to develop the model.

The main factors that change the small-strain property including  $\sigma'_o$ , FC, PI, and OCR according to the previous study on laboratory test data are first considered in the model development. The developed simple model that includes these parameters can be applied to estimate Vs of clay, silt, and sand. In addition, the change of Vs due to FC showed in the regression result agrees well with the fine correction recommended in the liquefaction potential analysis. However, the model underestimates the Vs of gravel as GC increases. A correction term based on the prediction residual is added on the original model to overcome the bias of underprediction. The corrected model (Eq. (8)) can predict Vs of gravel very well for different regions of Taiwan, and thus, is potential to be used for the other countries.

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