Prediction of permeability coefficient for Quaternary Sediments of Drava River using different empirical correlations

M. Pap
National University of Public Service, Baja, Hungary, pap.miklos@uni-nke.hu

A. Mahler
Budapest University of Technology and Economics, Budapest, Hungary, mahler@mail.bme.hu

ABSTRACT: Permeability coefficient is the most significant soil parameter in seepage calculations. It has been recognized that permeability of granular soils is strongly related to the grain size, thus numerous empirical correlations have been developed to estimate permeability using its grain size characteristics. In this study the empirical correlations proposed by Hazen (1911), Beyer (1964), Amer and Awad (1974), Carrier (2003) and Chapuis (2004) are evaluated and compared to laboratory measurement results. Quaternary Sediments of Drava are very typical in the Southern Transdanubia, Hungary. Due to the development of water management for backwaters of river Drava the permeability of these deposits became a major question for the water flow analysis in this area.

Keywords: permeability coefficient; grain size distribution; empirical equations

1. Introduction

The permeability coefficient (*k*) for soils is the most notable factor in groundwater seepage calculations and analysis. Determination of a reliable permeability coefficient is obligate to reliably calculate, model and evaluate seepage in porous medium. There are several methods such as in-situ tests, laboratory measurements and empirical correlations to assess the value of permeability. During the in-situ tests the accurate measurement of k is limited by the uncertainties in geometry of the investigated soil layer and in hydraulic boundary conditions. Moreover, it is difficult to acquire representative samples for the laboratory tests; the tested specimens are often limited in size thus those may not properly represent the entire layer on site. Because of these facts the application of empirical or semi-empirical equations is needed and it also empowers fast estimation of permeability.

Quaternary Drava deposits with varied composition are very typical in the Southern Transdanubia region. Due to the development of water management for backwaters of river Drava the permeability of these deposits became a major question for the water flow analysis in this area.

The permeability for a single fluid flow can be estimated using empirical equations, capillary models, statistical procedures, and hydraulic radius theories. Several empirical or semi-empirical equations are published in the literature to predict the permeability of porous medium [1-6]. There are simple models that use some characteristic pore diameter, while more sophisticated formulas take into account other parameters (e.g. void ratio, viscosity, coefficient of uniformity etc.) as well, which have large effect on permeability [6, 7]. The combined models include many parameters as such as the size of the pores, their tortuosity and their connectivity to consider the relationships between the flowrate and the porous space.

The equation developed by Hazen [8] has been used for a century. This empirical equation uses the effective diameter for predicting the permeability of saturated loose sand. The formula is the following:

$$k = C_H \cdot D_{10}^2 \tag{1}$$

where k is the permeability in cm/s, C_H is the Hazen empirical coefficient and D_{10} is the diameter at which 10% of the sample's mass is comprised of particles with a diameter less than this value (cm). Lot of ranges have been given in geotechnical textbooks for value of C_H. According to the literature the value varies between 1 and 1000 [3], but usually assumed to be equal to 100. In this recent study C_H=100 is utilized consequently. The formula's applicability is generally limited to the range of d=0.01-0.30 cm [3].

A densely cited equation is the Kozeny-Carman equation, which was suggested by Kozeny [9] and subsequently modified by Carman [1, 2]. This relation describes the permeability coefficient as a function of the void ratio, the specific surface and a factor to take into account the shape and tortuosity of channels:

$$k = \frac{\gamma}{\mu \cdot C \cdot S^2} \cdot \frac{e^3}{1+e} \tag{2}$$

where *k* is the permeability in cm/s, γ is the unit weight of permeant, μ is the viscosity of permeant, *C* is the Kozeny-Carman empirical coefficient, *S* is the specific surface area per unit volume of particles (1/cm) and *e* is the void ratio.

Amer and Awad [10] used the preceding equation and completed it with their experimental results. This relation contains the coefficient of uniformity (C_U) to predict the permebaility for coarse sands. The formula is written as follows:

$$k = C_1 \cdot D_{10}^{2.32} \cdot C_U^{0.6} \cdot \frac{e^3}{1+e} \tag{3}$$

where k is the permeability in cm/s, C_1 is a constant, which equal to 35, D_{10} is the effective grain size (mm), C_U is the coefficient of uniformity, and e is the void ratio.

The formula developed by Beyer [11] has took into account the uniformity coefficient as a function of permeability. Beyer's data suggest an inverse relationship between C_U and k. This equation does not consider porosity and most useful for materials poorly graded with heterogeneous distributions, with uniformity coefficient between 1 and 20, and effective grain size between 0.06 mm and 0.6 mm. The equation can be expressed as follows:

$$k = 60 \cdot \log\left(\frac{500}{C_U}\right) \cdot D_{10}^2 \quad (4)$$

where k is the permeability in cm/s, C_U is the coefficient of uniformity, and D_{10} is the effective grain size (mm).

The Kozeny-Carman equation is approximately valid for sands and is not valid for fine grained soils. However, the formula is not frequently used, because it is difficult to determine the soil specific surface that can be either measured or estimated [12]. For practical use, Carrier [3] modified the Eq. (2) by applying for calculation of the specific surface the effective diameter that can be determined using the grain size distribution curve. The equation describes the permeability in the following form:

$$k = 1.99 \cdot 10^4 \cdot \left(\frac{100\%}{\sum \frac{f_i}{D_{li}^{0.404} \cdot D_{si}^{0.595}}}\right)^2 \cdot \left(\frac{1}{SF}\right)^2 \cdot \frac{e^3}{1+e} \quad (5)$$

where k is the permeability in cm/s, f_i is the fraction of particles between two sieve seizes (%), D_{li} is the diameter seize of the larger sieve (cm), D_{si} is the diameter seize of the smaller sieve (cm), e is the void ratio and SF is the shape factor. The magnitude of SF may vary from between 6 and 8, depending on the angularity of the soil particles. In his recent study SF=7 is used for all soil types. This formula can be applied in silts, sands, and even gravelly sands [3, 12].

More recently, Chapuis [4] proposed an empirical relationship for the permeability coefficient. This equation is valid for natural, uniform sand and gravel to estimate the permeability coefficient that is in the range of 10^{-1}–10^{-3} cm/s. This can be extended to natural, silty sands without plasticity. It is not valid for crushed materials or silty soils with some plasticity [4, 12]. The equation uses the effective diameter and the void ratio:

$$k = 2.4622\left[D_{10}^2 \cdot \frac{e^3}{1+e}\right]^{0.7825} \quad (6)$$

where k is the permeability in cm/s, D_{10} is the effective size (mm) and e is the void ratio.

The scope of the study was to compare the empirical equations developed by Chapuis (2004), Carrier (2003), Amer and Awad (1974), Beyer (1964) and Hazen (1911) for prediction of the permeability coefficient of the deposits in backwater area of Drava River. Furthermore, the aim was to evaluate the feasibility of the recent formulas for these different soil types (e.g. sand, silty sand, sandy clayey silt) by comparing the predicted values of the permeability to the coefficients given by the laboratory measurements.

2. Materials and methods

2.1. Laboratory permeability test

In this study, the falling head permeability test based on the Hungarian standards was used to measure the permeability coefficient of soil samples. It was carried out in a falling head permeability device [13]. The permeability test involves seepage through a soil sample connected to a standpipe which provides the water head and allows measuring the volume of water flowing through the soil sample. The water starts to flow through the sample until the water in the standpipe reaches a given lower limit. The time required for the water in the standpipe to drop to the lower level is measured. Fig. 1 presents a schematic view of the measuring method.

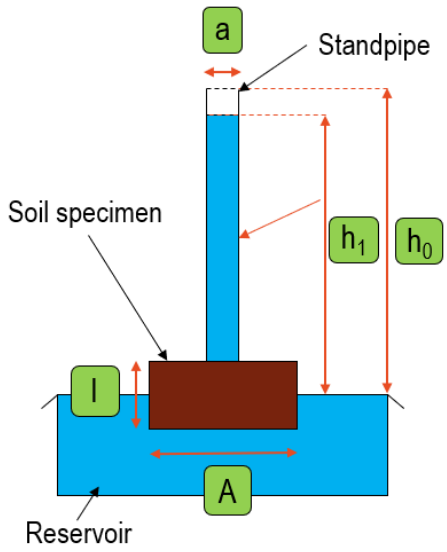

Figure 1. Falling head test device.

Based on the measurement results the permeability coefficient can be calculated with the following formula:

$$k = \frac{a \cdot l}{A \cdot \Delta t} \cdot \ln \frac{h_0}{h_1} \quad (7)$$

where k is the permeability coefficient in m/s, a is the cross section of the standpipe (m^2), A is the cross section of the soil sample (m^2), Δt is the measured time for the water column decreasing (s), h_0 is the initial water head (m), h_1 is the water head related to the recorded time (m).

2.2. Grain size distribution

The classification of Drava sediments was performed according to the European standards. The test consists of shaking the soil sample through a set of sieves that have progressively smaller openings. After the shaken, the mass of soil remained on each sieve is measured. The amount of the silt and clay particles is determined by hydrometer analysis [14].

More sophisticated semi-empirical equations take into account the void ratio to predict the permeability coefficient. Thus, the void ratios were determined for each specimen, and were used to calculate k values.

Based on the results of the grain size distribution and the hydraulic conductivity properties the soil samples were divided into three different categories. The first division is for the sand soils (Fig. 2). These samples include less than 15% fine-grain size (smaller than 0.063 mm) fraction, the void ratio varied between 0.54 and 0.81, and the value of effective grain size (d_{10}) was in range from 0.051 mm to 0.164 mm.

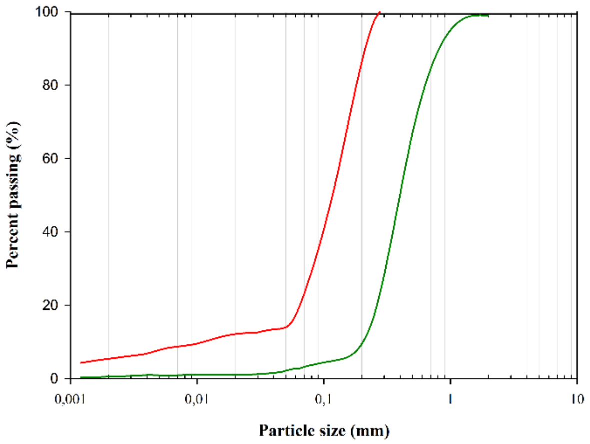

Figure 2. Grain size distribution curves of the sand

The silty sand and clayey sand soils belong to the second group. These soil specimens include ca. 16-39 % silt and clay content. The void ratio of these soils varied in a wide range from 0.49 to 2.24. The effective grain size is given between 0.003 mm and 0.028 mm.

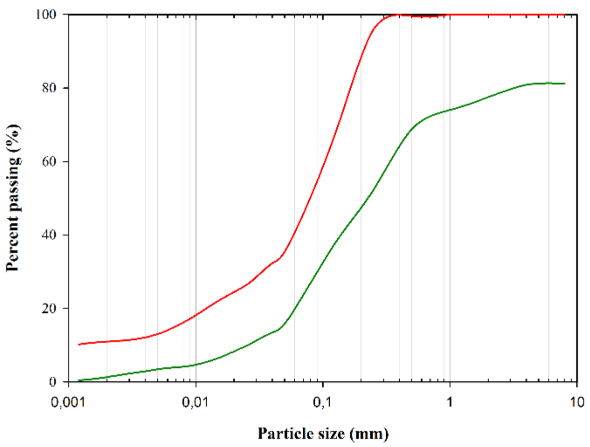

Figure 3. Grain size distribution curves of the silty sand

The third group of the samples is for the soils with cohesion. The sandy clayey silt soils contain ca. 41-78% fine particles and 10-21% sand size fraction. The value of the void ratio for the cohesive soils is given between 0.70 and 3.48, so the structure of these samples was very loose. However, the effective grain size varied in a narrow range from 0.002 mm to 0.012 mm.

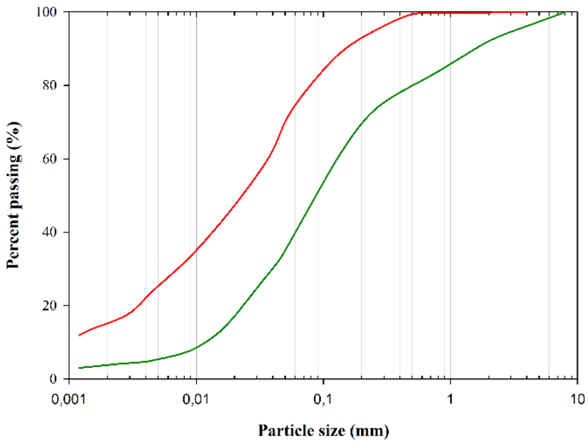

Figure 4. Grain size distribution curves of the sandy clayey silt

All considered, 50 different soil samples were measured and compared in this paper due to the different grain size distribution.

3. Results

The permeability coefficient of each sample was calculated based on the grain size distribution curves and void ratios. The results were compared to the ones acquired by falling head test. Fig. 5 to 9 show the measured and estimated k-values using equation developed by Carrier (2003), Chapuis (2004), Hazen (1911), Amer and Awad (1974), and Beyer (1964). On these figures, the linear function represents the perfect estimation (i.e. the measured and predicted values are equal to each other).

In most cases, all equation overestimated the permeability coefficient of the specimens measured. The formulas of Chapuis and Carrier provided mostly similar results generally for all soil type, while the correlation of other methods resulted in different values. Beyer's equation overestimated significantly the permeability coefficient, especially in case of sand. According to the figures the estimations of the Hazen formula seemed to give more realistic results for the all groups mentioned above.

According to all empirical equations analyzed the permeability coefficient of the sand soils were overestimated. The Carrier, Chapuis and Hazen formulas show very comparable results while Amer and Awad equation overestimates permeability by about an order of magnitude. Due to the scatter of data the Beyer form gave the less realistic outcome in the case of sand soils. The permeability coefficients were given with error of two order of magnitude at least, but the deviation was quite consistent. It seems in case of sand soils the calculated data were given in a narrow range to each equation used.

Silty sands show a different picture. The permeability of these soils were mostly overestimated, but the calculated data fits well to the mesured ones in the range of 10^{-6}-10^{-5} m/s. It can be observed, the lower the permeability, the higher the accuracy of the prediction methods in the case of silty sands. However, in the low permeability range the use of Hazen formula provided corresponding permeability to the mesured data.

For soils with plasticity, the trend is very similar to the silty sands. In the range of 10^{-8}-10^{-7} m/s the predictions are inaccurate using the methods of Carrier, Chapuis, Amer and Awad, and Beyer. Nevertheless, the results were obtained by Hazen equation show more accuracy in lower range than range of 10^{-6}-10^{-5} m/s comparing to the other empirical equations used.

The predicted and measured values in whole range tested were closer in the case of Hazen method. The Hazen equation did not resulted in a systematic deviation, but the scatter of the data more significant. Based on the measurements data concerning to cohesive soils fit appropriate with equality function considering that applicability of these equations is limited smaller fine contents. To be able to compare the different prediction methods considering the type of soils the mean squared error of the estimation and measured data was obtained. Values of the mean squared error are shown on Fig. 10.

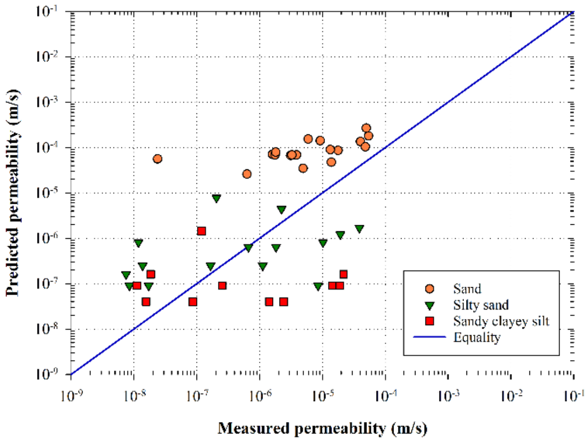

Figure 7. Measured versus predicted k-value using Hazen (1911) equation

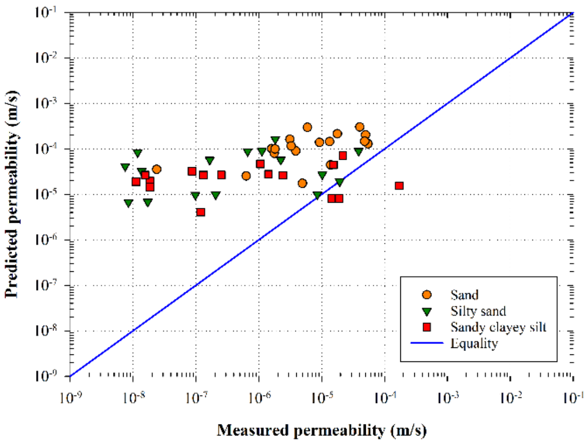

Figure 5. Measured versus predicted k-value using Carrier (2003) equation

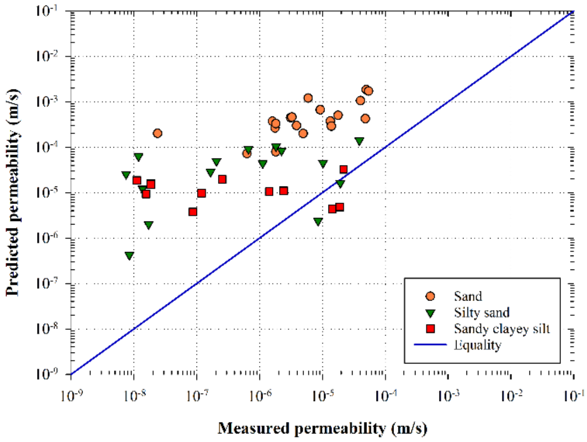

Figure 8. Measured versus predicted k-value using Amer and Awad (1974) equation

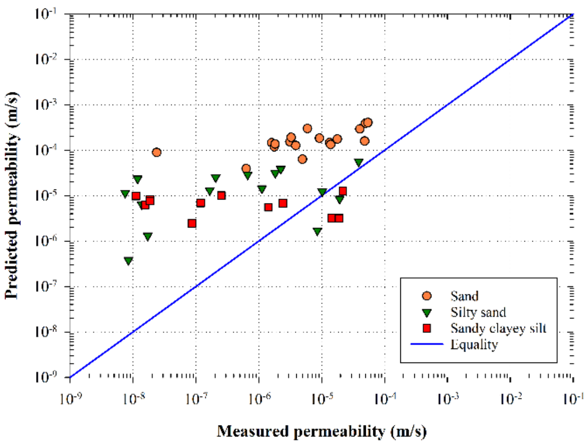

Figure 6. Measured versus predicted k-value using Chapuis (2004) equation

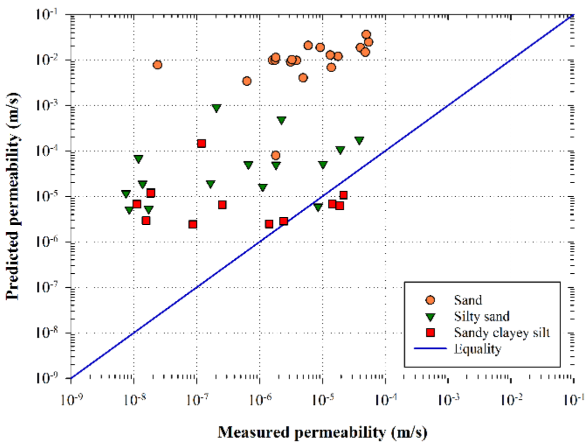

Figure 9. Measured versus predicted k-value using Beyer (1964) equation

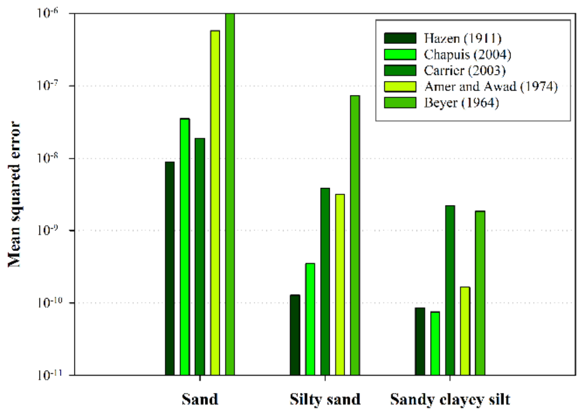

Figure 10. Mean squared error of the applied prediction methods

Based on the mean squared error the applicability and reliability of the methods can be assessed. It can be stated that the hydraulic conductivity of the sands was estimated with relatively small and very similar error by Hazen, Chapuis and Carrier formulas, despite of the fact that Beyer, Amer and Awad prediction methods should be appropriate to estimate permeability of sand as well. It implies that the use different empirical constants in this formulas may be useful for quaternary Drava sands. In the case of silty sand Hazen and Chapuis equation show similar errors, but the Hazen correlation gave the most accurate prediction. The permeability coefficient of the sandy clayey silt was estimated with similar accuracy by Amer and Awad, Hazen and Chapuis. Table 1 shows the most and less accurate equation for each soil type of Drava deposits tested.

Table 1. Most and less accurate formula for soil type tested

Soil type	Best	Worst
Sand	Hazen (1911)	Beyer (1964)
Silty sand	Hazen (1911)	Beyer (1964)
Sandy clayey silt	Chapuis (2004)	Carrier (2003)

4. Conclusion

In this paper, empirical correlations for prediction the permeability coefficients of different soils were compared and evaluated. These formulas use different methodologies to calculate the permeability coefficient using the grain size distribution curve. According to the literature, Hazen formula is suitable only for soils having particle sizes in the range of 0.01-0.30 cm, Chapuis equation is capable to estimate the permeability coefficient of uniform sand and gravel in the range of 10^{-3}–10^{-5} m/s, while the Carrier formula is approximately valid for sands and is not valid for clays. Beyer's equation are useful for poorly graded soils with effective grain size in the range of 0.06-0.6 mm. Method of Amer and Awad is valid for coarse sands. The classified soils were divided to three groups: sand, silty sand, and sandy clayey silt. For each sample the permeability coefficient was measured in the laboratory by the falling head permeability test and was also estimated using the empirical correlations mentioned above. The estimated and measured values were compared and evaluated.

Generally, most correlations sytematically overestimated the permeability coefficient, except Hazen form. Hazen, Carrier and Chapuis formula provided estimation of comparable accuracy for sand soils. Despite that the Carrier equation should be feasible for silt soils, it predicted the permeability with huge error for silty sands and sandy clayey silts. For all soil type, Beyer's equation provided less reliable results, while Amer and Awad method proved more useable for silty soils and soil with plasticity. The permeability coefficients of the specimens were overestimated consequently and significantly indicating that different empirical constants may be necessary when estimating the permeability of Drava deposits.

As a conclusion it can be stated the Hazen and Chapuis equation resulted in estimations closer to the measured values. The systematic deviations in case of some methods and soil types implies that different empirical constants may be necessary to estimate the permeability of Quaternary Danube soils in a more reliable way.

References

[1] Carman, P. C. "Fluid flow through granular beds", Transactions of the Institute of Chemical Engineering, 15, pp. 150-166, 1937. https://doi.org/10.1016/S0263-8762(97)80003-2

[2] Carman, P. C. "Flow of gases through porous media", Butterworths, London, UK, 1956.

[3] Carrier, W. D. "Goodbye, Hazen; Hello, Kozeny-Carman", Journal of Geotechnical and Geoenvironmental Engineering, 129(11), pp. 1054-1056, 2003. https://doi.org/10.1061/(ASCE)1090-0241(2003)129:11(1054)

[4] Chapuis, R. P. "Predicting the saturated hydraulic conductivity of sand and gravel using effective diameter and void ratio", Canadian Geotechnical Journal, 41(5), pp. 787-795, 2004. https://doi.org/10.1139/t04-022

[5] Chikhaoui, M., Belayachi, N., Nechnech, A., Hoxha, D. "Experimental Characterization of the Hydromechanical Properties of the Gypsum Soil of Sebkha of Oran", Periodica Polytechnica Civil Engineering, 61(4), pp. 706-717, 2017. https://doi.org/10.3311/PPci.10468

[6] Nagy, L. "Permeability of well graded soils", Periodica Polytechnica Civil Engineering, 55(2), pp. 199-204, 2011. https://doi.org/10.3311/pp.ci.2011-2.12

[7] Kieu, M., Mahler, A. "A Study on the Relationship Between Matric Suction and the Void Ratio and Moisture Content of a Compacted Unsaturated Soil", Periodica Polytechnica Civil Engineering", 62(3), pp. 709-716, 2018. https://doi.org/10.3311/PPci.11974

[8] Hazen, A. "Discussion of Dams on sand foundations by A. C. Koenig", Trans. Am. Soc. Civ. Eng., 73, pp. 199–203, 1911.

[9] Kozeny, J. "Über kapillare Leitung des Wassers im Boden", Akad. Wiss. Wien, 136, pp. 271-306, 1927.

[10] Amer, A. M., Awad, A. A. "Permeability of cohesionless soils", Journal of the Geotechnical Engineering Division, 100(12), pp. 1309-1316, 1974.

[11] Beyer, W., "Zur Bestimmung der Wasserdurchlässigkeit von Kiesen und Sanden aus der Kornverteilungskurve", Wasserwirtschaft und Technik, 14, pp. 165-169, 1964.

[12] Das, B. M. "Advanced soil mechanics", Taylor & Francis, New York, USA, 2008.

[13] Nagy, L., Takács, A., Huszák, T., Mahler, A., Varga, G., "Comparison of permeability testing methods", In: Proceedings of the 18th International Conference on Soil Mechanics and Geotechnical Engineering, Paris, pp. 399-402, 2013.

[14] Das, B. M. "Principles of Geotechnical Engineering", Thomson, Ontario, Canada, 2006.