Statistical uncertainty in evaluating strength of deep mixing column

Tsutomu Namikawa
Civil Engineering/Sibaura Institute of Technology, Tokyo, Japan, namikawa@shibaura-it.ac.jp

ABSTRACT: This paper presents the investigation of the statistical uncertainty involved in statistical parameters of unconfined compressive strength q_u of cored samples retrieved from cement-treated columns by deep mixing method. The statistical uncertainty is quantitatively evaluated by using a Bayesian approach. In the proposed approach, the mean μ_{qu}, variance σ^2_{qu}, and autocorrelation distance θ_{lnqu} are inferred as the statistical parameters of q_u. The posterior probability distribution of μ_{qu}, σ^2_{qu}, and θ_{lnqu} after observing core strength is defined by the Bayesian approach to inference. It is difficult to directly evaluate the probability quantities of the posterior probability distribution which is a joint probability density function of the three parameters. Thus, a Markov chain Monte Carlo (MCMC) method is adopted to draw the parameter values from their target posterior distributions and the probability quantities are evaluated from the drawn values of the parameters. The analysis is conducted to infer μ_{qu}, σ^2_{qu}, and θ_{lnqu} from the q_u values of cored samples retrieved from cement-treated columns in a practical project. The analysis results show that the some amount of the statistical uncertainty is involved in the evaluated values of μ_{qu}, σ^2_{qu}, and θ_{lnqu} from the core strength. The statistical sample size and the spatial correlation affect the amount of the statistical uncertainty. The analysis results provide useful information on core sampling plans in quality assurance procedures of the deep cement mixing method.

Keywords: statistical uncertainty; cement-treated soil; strength; Bayesian approach.

1. Introduction

In deep cement mixing method, the cement-treated soil columns are constructed by mixing in situ soils and cement at construction sites. Accordingly, the strength of the constructed columns varies spatially. Thus, it is very important to assure the quality of constructed cement-treated soil columns in the deep mixing method. In the quality assurance procedures of the deep mixing method, unconfined compressive strength q_u of cored sample retrieved from cement-treated soil columns is normally used as the property to verify the quality of the columns. In this process, the statistical parameters, e.g., the mean and variance, are evaluated from the core strength data. These evaluated parameters are sample statistical parameters, which involve the statistical uncertainty depending on the statistical sample size, i.e., the number of samples, and other factors. Therefore, the estimation of the statistical uncertainty is required to assure the quality of the ground improvement properly.

In this paper, the statistical uncertainty involved in the statistical parameters of the core strength is investigated by using a Bayesian approach. The mean μ_{qu}, variance σ^2_{qu}, and autocorrelation distance θ_{lnqu} are inferred as the statistical parameters of q_u. In the Bayesian approach to inference, the posterior probability distribution of μ_{qu}, σ^2_{qu}, and θ_{lnqu} is defined as a product a likelihood function in terms of observed q_u values and prior probability distributions of each parameter. Since the posterior probability distribution is defined as a joint probability density function of the three parameters, it is difficult to directly calculate this probability quantities. In this study, a Markov chain Monte Carlo (MCMC) method is adopted to draw the parameter values from their target posterior distributions; the probability quantities are evaluated from the drawn values of the parameters. The analysis is conducted to infer μ_{qu}, σ^2_{qu}, and θ_{lnqu} from the q_u values of cored samples retrieved from cement-treated columns in a practical project. The analysis results show quantitatively the statistical uncertainty involved in the evaluated values of μ_{qu}, σ^2_{qu}, and θ_{lnqu} from the core strength. The analysis results provide useful information on core sampling plans in quality assurance procedures of the deep cement mixing method.

2. Approach

A Bayesian inference approach proposed by Namikawa [1] is used in this study. In this approach, assuming the unconfined compressive strength q_u of cement-treated soils follows a multivariate normal distribution, the statistical parameters μ_{qu}, σ^2_{qu}, and θ_{lnqu} are estimated from a likelihood function and the prior probability distributions of each parameter. In this study, a multivariate lognormal distribution is adopted for the probability distribution for q_u.

2.1. Probability distribution of strength

q_u of cement-treated soils is assumed to follow a multivariate lognormal distribution as follows:

$$p\left(\mathbf{q_u}\middle|\mu_{qu},\sigma_{qu}^2,\theta_{lnqu}\right)=\frac{1}{\sqrt{(2\pi)^n\left(\sigma_{lnqu}^2\right)^n|\mathbf{C}|\prod_{i=1}^n q_u(\mathbf{r_i})}}\exp\left\{-\frac{1}{2\sigma_{lnqu}^2}\left(\mathbf{lnq_u}-\boldsymbol{\mu}_{\mathbf{lnqu}}\right)^T\mathbf{C}^{-1}\left(\mathbf{lnq_u}-\boldsymbol{\mu}_{\mathbf{lnqu}}\right)\right\} \tag{1}$$

$$\mathbf{lnq_u}=\begin{bmatrix}\ln q_u(\mathbf{r_1})\\ \vdots\\ \ln q_u(\mathbf{r_n})\end{bmatrix},\quad \boldsymbol{\mu}_{\mathbf{lnqu}}=\begin{bmatrix}\mu_{lnqu}\\ \vdots\\ \mu_{lnqu}\end{bmatrix},$$

$$\mathbf{C}=\rho_{qu}(\mathbf{d})=\exp\left(-\frac{[\mathbf{r_i}-\mathbf{r_j}]}{\theta_{lnqu}}\right)$$

where n is the number of q_u values, $\mathbf{r_i}$ is the space vector at the point i, μ_{lnqu} is the mean of ln q_u, σ^2_{lnqu} is the variance of ln q_u, and θ_{lnqu} is the autocorrelation distance of ln q_u. In this equation, an exponential type autocorrelation function is assumed for the spatial variability of ln q_u. μ_{lnqu} and σ^2_{lnqu} are expressed in terms of μ_{qu} and σ^2_{qu}:

$$\mu_{lnqu}=\ln\mu_{qu}-\frac{1}{2}\ln\left\{1+\frac{\sigma_{qu}^2}{\left(\mu_{qu}\right)^2}\right\} \tag{2}$$
$$\sigma_{lnqu}^2=\ln\left\{1+\frac{\sigma_{qu}^2}{\left(\mu_{qu}\right)^2}\right\}$$

Conversely, μ_{qu} and σ^2_{qu} are described as,

$$\mu_{qu}=\exp\left\{\mu_{lnqu}+\frac{\sigma_{lnqu}^2}{2}\right\} \tag{3}$$
$$\sigma_{qu}^2=\exp\left\{2\mu_{lnqu}+2\sigma_{lnqu}^2\right\}-\exp\left\{2\mu_{lnqu}+\sigma_{lnqu}^2\right\}$$

In the Bayesian inference approach, realizations of μ_{lnqu}, σ^2_{lnqu} and θ_{lnqu} are drawn from the posterior probability distribution. Thereafter, μ_{qu} and σ^2_{qu} are calculated from the realizations of μ_{lnqu} and σ^2_{lnqu} by Eq. (3). It should be noted that the statistical uncertainty of θ_{lnqu} is examined as the parameter of the autocorrelation distance.

2.2. Bayesian inference approach

In the Bayesian inference approach, the posterior probability distribution $p(\mu_{lnqu}, \sigma^2_{lnqu}, \theta_{lnqu}|\mathbf{q_u})$ after observing data is described as the production of the likelihood function $p(\mathbf{q_u}|\mu_{lnqu}, \sigma^2_{lnqu}, \theta_{lnqu})$ and the prior probability distributions of each parameter $p(\mu_{lnqu})$, $p(\sigma^2_{lnqu})$, $p(\theta_{lnqu})$. The posterior probability distribution is given by

$$p\left(\mu_{lnqu},\sigma_{lnqu}^2,\theta_{lnqu}\middle|\mathbf{q_u}\right)\propto p\left(\mathbf{q_u}\middle|\mu_{lnqu},\sigma_{lnqu}^2,\theta_{lnqu}\right)p\left(\mu_{lnqu}\right)p\left(\sigma_{lnqu}^2\right)p\left(\theta_{lnqu}\right) \tag{4}$$

The posterior distribution is expressed as a joint probability density function among μ_{lnqu}, σ^2_{lnqu}, and θ_{lnqu}.

2.3. Markov chain Monte Carlo method

It is difficlut to directly calculate the probability propeties of the joint probability distribution expressed in Eq. (4). Namikawa [1] adopted Markov chain Monte Carlo (MCMC) method to evalaute the probability quantities of the statistical parameters. In the MCMC method, the realization values of μ_{lnqu}, σ^2_{lnqu}, and θ_{lnqu} are drawn from conditional distributions and evaluate the probability quantities of these parameters. The used MCMC method is briefly described here.

The normal distribution is selected for the prior distribution of μ_{lnqu}. Accordingly, the prior distribution becomes the natural conjugate distribution and the conditional posterior distribution of μ_{lnqu} is described as,

$$p\left(\mu_{lnqu}\middle|\sigma_{lnqu}^2,\theta_{lnqu},\mathbf{q_u}\right)\propto p\left(\mathbf{q_u}\middle|\mu_{lnqu},\sigma_{lnqu}^2,\theta_{lnqu}\right)p\left(\mu_{lnqu}\right)\propto\exp\left\{-\frac{\left(\mu_{lnqu}-\mu_\mu\right)^2}{2\sigma_\mu^2}\right\} \tag{5}$$

$$\mu_\mu=\frac{2\mu_{\mu0}\sigma_{lnqu}^2+\sigma_{\mu0}^2\sum_{i=1}^n\sum_{j=1}^n\zeta_{ij}\left(q_{ui}+q_{uj}\right)}{2\left(\sigma_{lnqu}^2+\sigma_{\mu0}^2\sum_{i=1}^n\sum_{j=1}^n\zeta_{ij}\right)}$$

$$\sigma_\mu^2=\left(\frac{1}{\sigma_{\mu0}^2}+\frac{\sum_{i=1}^n\sum_{j=1}^n\zeta_{ij}}{\sigma_{lnqu}^2}\right)^{-1}$$

where $\mu_{\mu0}$ and $\sigma^2_{\mu0}$ are the mean and variance of the prior distribution of μ_{lnqu}, ζ_{ij} is the elements of $\mathbf{C}^{-1}$, and q_{ui} is q_u at the point i. The realizations of μ_{lnqu} is directly drawn from the conditional posterior distribution defined in Eq. (5) by using a Gibbs sampler.

The inverse gamma distribution is selected for the prior distribution of σ^2_{lnqu}. Accordingly, the prior distribution becomes the natural conjugate distribution and the conditional posterior distribution of σ^2_{lnqu} is described as,

$$p\left(\sigma_{lnqu}^2\middle|\theta_{lnqu},\mu_{lnqu},\mathbf{q_u}\right)\propto p\left(\mathbf{q_u}\middle|\mu_{lnqu},\sigma_{lnqu}^2,\theta_{lnqu}\right)p\left(\sigma_{lnqu}^2\right)$$
$$\propto\left(\sigma_{lnqu}^2\right)^{-(\alpha_\sigma+1)}\exp\left\{-\frac{\beta_\sigma}{\sigma_{lnqu}^2}\right\} \tag{6}$$
$$\alpha_\sigma=\frac{n}{2}+\alpha_{\sigma0}$$
$$\beta_\sigma=\beta_{\sigma0}+\frac{1}{2}\left(\mathbf{lnq_u}-\boldsymbol{\mu}_{\mathbf{lnqu}}\right)^T\mathbf{C}^{-1}\left(\mathbf{lnq_u}-\boldsymbol{\mu}_{\mathbf{lnqu}}\right)$$

where $\alpha_{\sigma0}$ and $\beta_{\sigma0}$ are the parameters of the inverse gamma distribution of σ^2_{lnqu}. The realizations of σ^2_{lnqu} is directly drawn from the conditional posterior distribution defined in Eq. (6) by using the Gibbs sampler.

The normal distribution is selected for the prior distribution of θ_{lnqu}. The prior distribution is not the natural conjugate distribution. The conditional posterior distribution of θ_{lnqu} is described as,

$$p\left(\theta_{lnqu}\middle|\mu_{lnqu},\sigma_{lnqu}^2,\mathbf{q_u}\right)\propto p\left(\mathbf{q_u}\middle|\mu_{lnqu},\sigma_{lnqu}^2,\theta_{lnqu}\right)p\left(\theta_{lnqu}\right)\propto\frac{1}{\sqrt{|\mathbf{C}|}}\exp\left\{-\frac{1}{2\sigma_{qu}^2}\left(\mathbf{lnq_u}-\boldsymbol{\mu}_{\mathbf{lnqu}}\right)^T\mathbf{C}^{-1}\left(\mathbf{lnq_u}-\boldsymbol{\mu}_{\mathbf{lnqu}}\right)\right\}\exp\left\{-\frac{\left(\theta_{lnqu}-\mu_{\theta0}\right)^2}{2\sigma_{\theta0}^2}\right\} \tag{7}$$

where $\mu_{\theta 0}$ and $\sigma^2{}_{\theta 0}$ are the mean and variance of the prior distribution of θ_{lnqu}. The realizations of θ_{lnqu} is drawn from the conditional posterior distribution defined in Eq. (7) by using a Metropolis-Hastings algorithm.

In this study, the Gibbs sampling and the Metropolis-Hastings algorithm are used in a combination to sample from the joint posterior probability distribution. Refer to Namikawa [1] for a detailed description of the MCMC process to draw the realization values.

3. Analysis of core strength data

3.1. Core strength data

The statistical uncertainty involving the estimated values of μ_{qu}, $\sigma^2{}_{qu}$, and θ_{lnqu} from core strength data is invetigated by using the Bayesian inference framework desribed in the previous section. Assuming that q_u follows a multivariate normal distribution, Namikawa [1] has adopted a core strength data reported by Namikawa and Koseki [2] to estimate the statistical uncertainty for μ_{qu}, $\sigma^2{}_{qu}$, and the autocorrelation distance of q_u, θ_{qu}. In this study where q_u is assumed to follow the multivariate lognormal distribution, the same core sample data is used in the Bayesian inference approach. The influence of the probability distribution assumed for q_u on the inferred statistical uncertainty will be examined by comparing the infereance results in Namikawa [1] and this study.

Namikawa and Koseki [2] investigated the autocorrelation distance of the cored sample strength retrieved from the deep mixing columns with cement slurry. At that project site, the lattice-shaped improved ground was adopted to prevent the sand deposits from liquefaction. The No. 1 and No. 2 cored sample data reported by Namikawa and Koseki [2] are adopted as an observation data in this study. The distribution of the cored sample strength is shown in Fig. 1. The soil profile at the location consists of upper sand and lower clay layers. The data were divided into two groups: the cement-treated sand portions (1-S and 2-S) and the cement-treated clay portions (1-C and 2-C). Fig. 1 shows the strength of the cement-treated soil does not increase with the depth, indicating that it is difficult to distinguish between the trend and autocorrelation influences on the core strength distribution. Therefore, the trend along the depth is not considered in this study.

The sample mean $s\mu_{\mathrm{qu}}$ and variance $s\sigma^2{}_{\mathrm{qu}}$ of q_{u} are shown in Table 1. The q_{u} values of the cored samples retrieved from the single core boring hole are used in 1-S, 2-S, 1-C, and 2-C, whereas the q_{u} values of the cored samples retrieved from both the No. 1 and No. 2 boring holes are utilized in 12-S and 12-C. The sample statistical parameteres of ln q_u are also shown in Table 1. The sample autocorrelation distance of ln q_u, $s\theta_{lnqu}$, in the vertical direction is calculated by the maximum likelihood method. Since the separation distance between the two borings is greater than 80 m, any horizontal correlation between the strength values at the two borings is neglected for 12-S and 12-C.

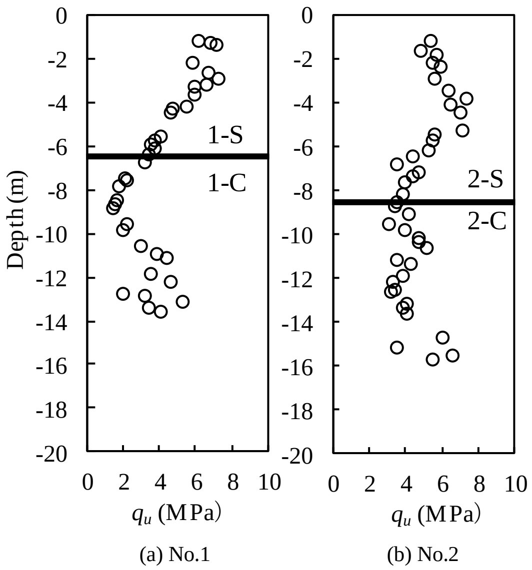


Figure 1. Profile of q_u of cored samples

Table 1. Statistical quantities of the cored sample strength

Group	Boring	n	$s\mu_{qu}$	$s\sigma^2{}_{qu}$	$s\theta_{lnqu}$
1-S	No. 1	17	5.39 MPa	1.81	2.7 m
2-S	No. 2	20	5.43 MPa	1.21	1.2 m
12-S	No. 1, No. 2	37	5.41 MPa	1.44	1.8 m
1-C	No. 1	19	2.92 MPa	1.37	0.6 m
2-C	No. 2	21	4.20 MPa	0.910	0.3 m
12-C	No. 1, No. 2	40	3.59 MPa	1.52	0.9 m

Note: n = sample size; $s\mu_{qu}$ = sample mean; $s\sigma^2{}_{qu}$ = sample variance; $s\theta_{lnqu}$ = sample autocorrelation distance for ln q_u

3.2. Parameters for prior distribution

The parameter values of the prior distribution are determined from the statistical data of the cored sample strength in past studies (Saitoh et al. [3]; Namikawa [4]; Namikawa and Koseki [2]). Using Eq. (2), μ_{lnqu} and $\sigma^2{}_{lnqu}$ are calculated from μ_{qu} and $\sigma^2{}_{qu}$ at the projects reported by Saitoh et al. [3]. The mean and variance of μ_{lnqu} are 1.14 and 0.390. The mean and variance of $\sigma^2{}_{lnqu}$ are 0.0657 and 0.000664; $\alpha_{\sigma 0}$ and $\beta_{\sigma 0}$ are set to be 8.50 and 0.493. Assuming that there is not a significant difference between the θ_{qu} and θ_{lnqu} values, θ_{lnqu} is determined based on θ_{qu}. According to Namikawa [1], the parameter values of the prior distribution of θ_{lnqu} are set to be 1 m and 1 ($\mu_{\theta 0}$ = 1 m and $\sigma^2{}_{\theta 0}$ = 1).

3.3. Simulation results

The 11000 values of the statistical parameters μ_{qu}, $\sigma^2{}_{qu}$ and θ_{lnqu} were sampled by the MCMC method. The sampled values for 1-S and 1-C are shown in Figs. 2 and 3. In the both cases, the realization values of μ_{qu}, $\sigma^2{}_{qu}$ and θ_{lnqu} varies significantly, indicating the large amount of the uncertainty included in the evaluated statistical parameters with the sample size of around 20.

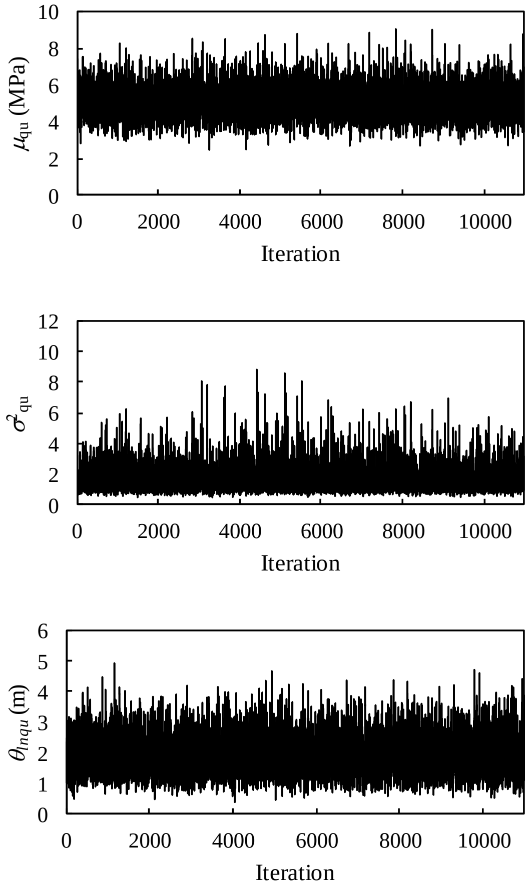


Figure 2. Realization values of μ_{qu}, σ^2_{qu} and θ_{lnqu} for 1-S

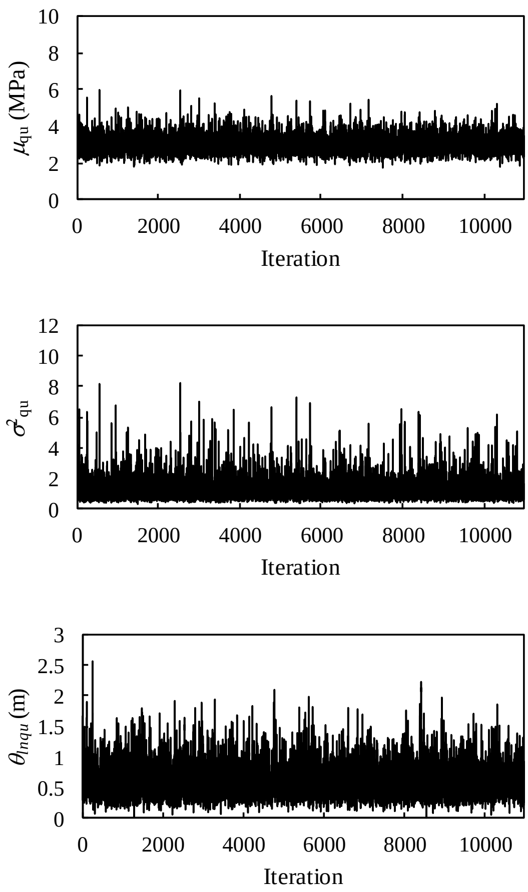


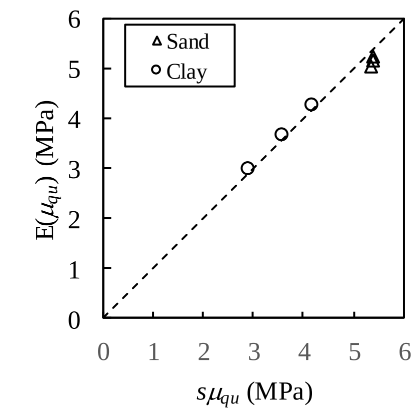
Figure 3. Realization values of μ_{qu}, σ^2_{qu} and θ_{lnqu} for 1-C

The statistical quantities of the drawn values of the parameters are summarized in Table 2. The comparison between the sample statistical values and the statistical quantities of the drawn μ_{qu}, σ^2_{qu} and θ_{lnqu} values are shown in Fig. 4. While Fig. 4 (a) shows that the mean of μ_{qu}, E(μ_{qu}), corresponds to $s\mu_{\mathrm{qu}}$, there is some difference between the mean of σ^2_{qu}, E(σ^2_{qu}), and $s\sigma^2_{\mathrm{qu}}$. The mean of θ_{lnqu} also differs from $s\theta_{lnqu}$ in some cases. This discrepancy may be caused by a correlation between the statistical parameters.

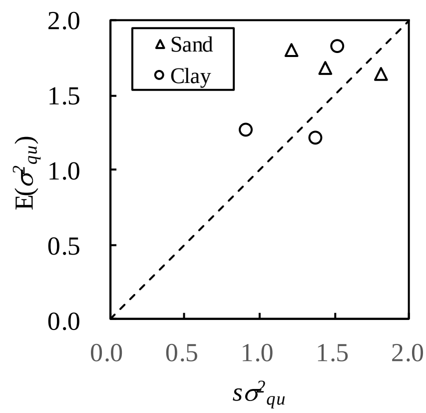
Table 2. Statistical quantities of drawn parameters

Group	Statistical quantities	μ_{qu}	σ^2_{qu}	θ_{lnqu}
1-S	Mean	5.06 MPa	1.64	1.99 m
	SD	0.791MPa	0.783	0.655 m
2-S	Mean	5.26 MPa	1.80	1.68 m
	SD	0.734 Mpa	0.830	0.609 m
12-S	Mean	5.17 MPa	1.69	1.90 m
	SD	0.554 MPa	0.599	0.558 m
1-C	Mean	3.00 MPa	1.21	0.630 m
	SD	0.431 Mpa	0.668	0.284 m
2-C	Mean	4.31 MPa	1.27	0.572 m
	SD	0.433 MPa	0.592	0.342 m
12-C	Mean	3.69 MPa	1.83	0.839 m
	SD	0.418 MPa	0.841	0.301 m

Note: SD = standard deviation; μ_{qu} = mean; σ^2_{qu} = variance; θ_{lnqu} = autocorrelation distance for ln q_u



(a) Mean

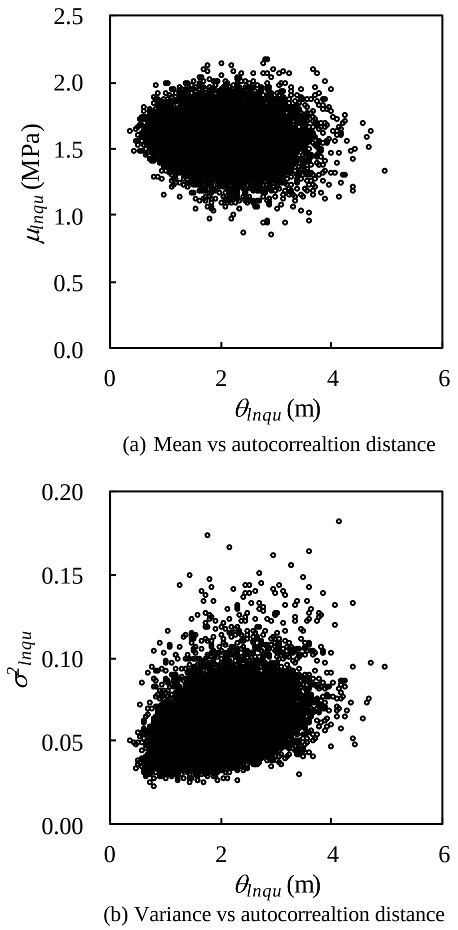


(b) Varaince

Figure 4. Comparison between sample statistical parameters and mean of drawn statistical parameters

The correlation between the statistical parameters for 1-S is shown in Fig. 5. Fig. 5(a) shows that there is no correlation between the drawn μ_{lnqu} and θ_{lnqu}. Coversely, Fig. 5(b) shows that the drawn σ^2_{qu} correlate positively with θ_{lnqu}. When θ_{lnqu} increases, the number of independent data sampled from the limited space decreases. Accordingly, the variability of the inferred values of σ^2_{lnqu} increases with the variability of θ_{lnqu}. This positive correlation may causes the variability of the ratio of E(σ^2_{qu}) to $s\sigma^2_{qu}$ in Fig. 4(b).

The coefficient of variance of the inferred μ_{qu}, σ^2_{qu} and θ_{lnqu} is summarized in Fig. 6. The coefficient of variance is calculated by dividing the standard deviaiton by the mean shown in Table 2 and represents the variability of the inferred parameters. Fig. 6 shows that the coefficient of variances of σ^2_{qu} and θ_{lnqu} are larger than that of μ_{qu}. The correlation between σ^2_{qu} and θ_{lnqu} may induce the larger variability of these drawn values. The coefficient of variances in the cases with the two boring data (12-S and 12-C) are lower than those in the cases with the one boring data (1-S, 2-S, 1-C and 2-C), indicating that the variability of the estimated statistical parameters can be reduced by increasing the number of the borings.



(a) Mean vs autocorrealtion distance

(b) Variance vs autocorrealtion distance

Figure 5. Correlation between drawn statistical parameters for 1-S

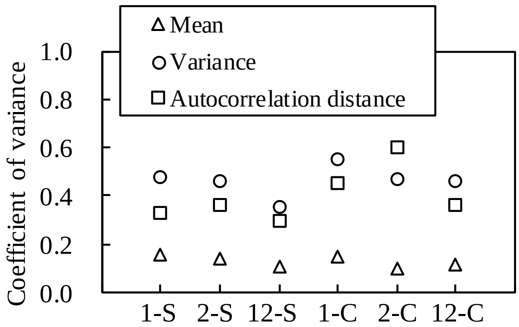


Figure 6. Coefficient of variance of drawn statistical parameters

3.4. Influence of probability distribution for q_u

In this study, q_u is assumed to be characterized statistically by a lognormal distribution. The author has assumed that q_u follows a normal distribution in the past study (Namikawa [1]). The influence of the probability distribution for q_u on the inferred statistical parameters is examined by comparing between the statistical quantities of the parameters based on the normal and lognormal distributions.

Before comparing between the Bayesian inference results, the goodness of fit for the normal and lognormal distributions against the data shown in Fig. 1 is examined by the Kolmogorov-Smirnov (K-S) test. In the K-S test, the D statistic is used as an indicator of the good fit. The D statistic is defined as,

$$D = \max|F(x) - S_n(x)| \tag{8}$$

where $S_n(x)$ is sample cumulative distribution function and $F(x)$ is target cumulative distribution function with the sample mean and variance. Low values of the D statistics indicates a better fit of the data to the distribution than do high values. The calculated D statistics for the two distributions is shown in Table 3. The D statistics supports the lognormal distribution for 1-C and 2-C. For the other data, the normal distribution is suitable to represent the probability distribution of q_u.

Table 3. D statistics

Group	Normal	Lognormal
1-S	0.148	0.182
2-S	0.096	0.135
12-S	0.091	0.131
1-C	0.206	0.167
2-C	0.190	0.156
12-C	0.104	0.175

The mean and variance estimated by the Bayesian approach on the assumption of the two distributions are compared in Fig. 7. In all cases, E(μ_{qu}) on the lognormal distribution agrees with that on the normal distribution, indicating that the probability distibution assumed for q_u does not significantly affect E(μ_{qu}) estimated in the Bayesian inference framework.

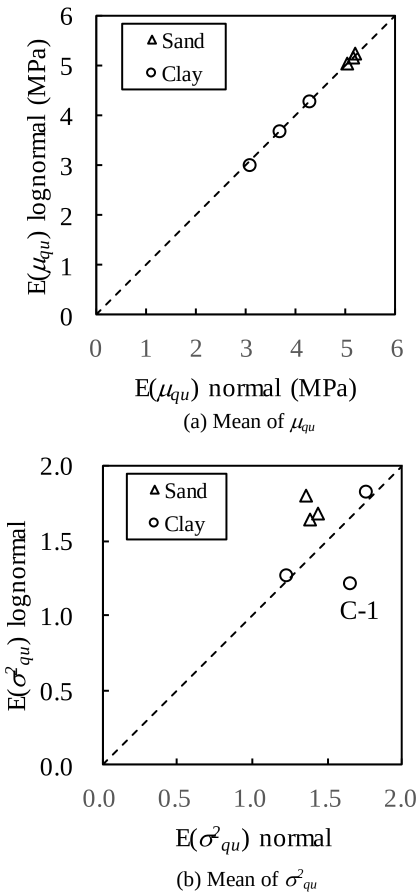


Figure 7. Comparison between mean of drawn statistical parameters on assumption of normal and lognormal distributions

Fig. 7(b) shows that E(σ^2_{qu}) on the lognormal distribution differs from that on normal distirbuton in some cases. In the cement-treated sand cases, the E(σ^2_{qu}) values on the lognormal distribution is larger than the E(σ^2_{qu}) values on the normal distibution. Table 3 indicates that the normal distibution is suitable for the probability distribution of q_u in the cement-treated sand cases. Fig. 7(b) also shows that, in the C-1 case, the E(σ^2_{qu}) values on the lognormal distribution is lower than the E(σ^2_{qu}) values on the normal distibution. In this case, the lognormal distibution is suitable for the probability distribution of q_u. These results indicate that the probability distibution assumed for q_u affects E(σ^2_{qu}) estimated in the Bayesian inference framework. The selection of probability distibution for q_u is important when estimating accurately the statistical parameters from the core strength data.

4. Conclusions

This paper presented the statistical uncertainty involved in the statistical parameters of q_u of the cement-treated ground improvement by deep mixing method. The Bayesian approach was used to evaluate the statistical uncertainty in the evaluation of the mean μ_{qu} and variance σ^2_{qu} of q_u, and autocorrelation distance θ_{lnqu} of ln q_u. The Markov chain Monte Carlo (MCMC) method was adopted to draw the parameter values from their target posterior distributions; the probability quantities were evaluated from the drawn values of the parameters. The analysis was conducted to infer μ_{qu}, σ^2_{qu}, and θ_{lnqu} from the q_u values of cored samples retrieved from cement-treated columns in a practical project. The analysis results showed quantitatively the statistical uncertainty involved in the evaluated values of μ_{qu}, σ^2_{qu}, and θ_{lnqu} from the core strength. In the analysis results, there is some difference in variance and autocorrelation distance between the sample and estimated statistical parameters. The correlation between σ^2_{qu} and θ_{lnqu} may induce such discrepancy between sample and estimated values. The influence of the probability distribution assumed for q_u on the statistical uncertainty was examined by comparing this study result with the past study result; the lognormal and normal distribution were assumed for q_u in this study and the past study. The comparison indicated the probability distibution assumed for q_u affects σ^2_{qu} estimated in the Bayesian inference framework.

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References

[1] Namikawa, T. "Evaluation of statistical uncertainty on cement-treated soil strength using Bayesian approach", Soils and Foundations, 59, 2019, pp. 1228-1240. https://doi.org/10.1016/j.samdf. 2019.04.010

[2] Namikawa, T., Koseki, J. "Effects of spatial correlation on the compression behavior of a cement-treated column", Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 139(8), 2013, pp. 1346–1359. https://doi.org/10.1061/(ASCE)GT.1943-5606.0000850

[3] Saitoh, S., Suzuki, Y., Nishioka, S., Okumura, R. "Required strength of cement improved ground", Proceedings, IS-Tokyo '96\The 2nd International Conference on Ground Improvement Geosystems, Tokyo, Balkema, Leiden, Netherland, 1996, pp. 557–562.

[4] Namikawa, T. "Characterization of strength variability of cement-treated soil column", Proceedings of the 4th Central Asian Geotechnical Symposium, Samarkand, 2012, pp. 172-177.